EASTERN UNIVERSITY, SRI LANKA

FIRST YEAR EXAMINATION IN SCIENCE - 2016/2017

FIRST SEMESTER(Aug./Sep., 2018)

MT 1222 - MATHEMATICAL SOFTWARE

Answer all questions

Time: Three hours

MATHEMATICA

1. (a)
   i. Compute numerical approximations to the square root and cube root of 10 accurate to 20 significant digits.
   ii. Determine the integer closest to $\sqrt{159}$.
   iii. Select a random number x, between 0 and 1 and compute $\sin^2 x + \cos^2 x$.
   iv. Approximate the sum $\frac{1}{15} + \frac{1}{17} + \frac{1}{19} + ... + \frac{1}{51}$.
   v. Print all numbers from 1 to 20, which are not multiples of 2, 3, and 5.

   [5 Marks]

   (b) Create a 5×5 zero matrix.
   i. Set the second column as $\{1, 2, 3, 4, 5\}$.
   ii. Set the third column as all entry 3.
   iii. Add a new row range from 10 to 14.

   [6 Marks]

(c) Consider the list $\{a, b, c, d, e, f, g, h, i\}$.

i. Insert an element p at the fourth position.

ii. Replace the elements at position three and seven by 2 and 3.

iii. Place x at prime-numbered positions. Note that the position is be primality, not for value.

(d) Compute the values of the first ten derivatives of $f(x) = e^{x^2}$ at $x = 0$ and g in tabular form.

Q2.(a) Sketch the graphs of the functions $y = -x^2$, $y = x^2$ and $y = x^2 \sin\left(\frac{1}{x}\right)$ o [0.02, 0.02] on one set of axes.

(b) Let P be a point at a distance a from the center of a circle of radius r. The by P as the circle rolls along a straight line is called a trochoid. Its parame are $x = r\theta \sin\theta$, $y = r - a\cos\theta$.

i. Sketch the trochoid with $r = 1$, $a = \frac{1}{2}$ as the circle makes four revolu

ii. What would the graph look like if $r = 1$, $a = 2$ so that the point i circle?

(c) Plot the given function, which is parameterized by the following equations

$$x(t) = \cos t - \cos 100t \sin t,$$

$$y(t) = 2\sin t - \sin 100t.$$

(d) Consider the range $0 \le t \le 2\pi$. If p dollars are compounded n times p annual interest rate of r, the money will be worth $p\left(1 + \frac{r}{n}\right)^{nt}$ dollars after much will the money be worth after t years if it is compounded continuou

(e) Given $f(x)$ whose graph is C, the slope of the line tangent to C at a $f(x) = \sin x$. Sketch the graph and its tangent line at $a = \frac{\pi}{3}$.

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MAPLE

(a) Write down appropriate *Maple* commands to

i. find the floating approximation of $e^{\frac{\sqrt{163}}{3}\pi}$ using a precision of 30 digits;

ii. solve $x^3 - 2.01x^2 - 4.415x + 3.2886 = 0$, correct to 15 decimal places;

iii. simplify the expression $\dfrac{5}{6(4x-1)} + \dfrac{9}{2(2x+1)} - \dfrac{3}{1-15x^2}$;

iv. compute $(AB)^2$, if $A = \begin{pmatrix} 1 & -1 & -2 \\ 0 & 1 & 2 \\ 1 & -3 & -2 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 1 & 2 \\ -1 & -1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$.

[15 Marks]

(b) Let $f(x) = 5x^2 - 3x + 10$. Plot $f(x)$ with the following options:

i. points should be cross;

ii. number of points 55;

iii. add a frame to the graph;

iv. graph colour green;

v. title "GRAPH" with font Courier size 16;

vi. axes labeled with font *Bradway* with font size 20;

vii. indicate a label $f(x) = 5x^2 - 3x + 10$ at the point $(2, 100)$.

[20 Marks]

The rate of growth of a population of insects in a certain habitat, $r(t)$, measured in thousands of insects per month is given by $r(t) = 10\, e^{\left(-\frac{3t}{100}\right)} \cos\left(\frac{\pi t}{6} - 3.5\right)$, where t is measured in months since January 1, 2018. Assume that there are 40,000 insects initially.

(a) find a function $p(t)$ which gives the size of the population at time t;

(b) plot $p(t)$ and $r(t)$ in one coordinate system for t between 0 and 24 with different colours;

(c) when is the population of insects minimal and when maximal during the two year period?

(d) what are the minimal and maximal values of the population? [15 Marks]