EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

FIRST EXAMINATION IN SCIENCE - 2016/2017

FIRST SEMESTER (August/ September, 2018)

MT 1012 - FOUNDATION OF MATHEMATICS

Answer all questions **Time : Two hours**

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1. (a) Using truth tables, show that $p \rightarrow q \equiv \sim p \vee q$.

   Hence, rewrite the compound statement $(\sim p \vee q) \longrightarrow (r \vee \sim q)$ using only the connectives $\wedge$ and $\sim$, where p, q and r are statements. [25 Marks]

   (b) Prove the following equivalences using the laws of logic:

   i. $p \wedge (p \vee q) \equiv p$;

   ii. $\sim (q \vee p) \vee (\sim p \wedge q) \equiv \sim p$,

   where p and q are statements. [30 Marks]

   (c) Using the valid argument forms, deduce the conclusion f from the premises given below:

$$l \rightarrow \sim k$$

$$e \rightarrow k$$

$$l$$

$$e \vee f$$

$$o \rightarrow g,$$

   where e, f, g, k, l and o are statements. [45 Marks]

2. (a) Let A, B and C be subsets of a set X. Simplify the expression:

$[(A \cup \Phi) \cap (A' \cup B) \cap (A \cup B \cup X)]'$. [2

(b) For any sets A and B, prove that $A \triangle B = (A \cup B) \setminus (A \cap B)$.

Hence show that:

i. $A \triangle B$ and $A \cap B$ are disjoint,

ii. $A \cup B = (A \triangle B) \cup (A \cap B)$.

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(c) For any set A, B and C, prove that $A \times (B \setminus C) = (A \times B) \setminus (A \times C)$. [2

3. (a) Let R be a relation defined on $\mathbb{C} \setminus \{0\}$ by $z_1 R z_2 \iff |z_1|(|z_2|^2 + 1) = |z_2|($

Prove that R is an equivalence relation .

If r is a real number such that $0 < r < 1$, then show that the equivalence

consists of two concentric circles with center at $(0,0)$ and radius r and $1/$

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(b) Let λ and μ be equivalence relations on a set A.

Prove that $\lambda \cap \mu$ is an equivalence relation.

Is $\lambda \cup \mu$ an equivalence relation? Justify your answer. [

(c) Prove that every partially ordered set has at most one last element. [

4. (a) Define the following terms:

i. *injective mapping*, ii. *surjective mapping*, iii. *inverse mapp*

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(b) Let a function $f : [-1, \infty) \to [-1, \infty)$ be defined by $f(x) = x^2 + 2x$. Show

bijective, and find f^{-1}. [

(c) Let $f : X \to Y$ be a mapping and A and B be any subsets of X. Prove th

f is injective if and only if $f(A \cap B) = f(A) \cap f(B)$. [