EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

FIRST EXAMINATION IN SCIENCE - 2016/2017

FIRST SEMESTER (August/ September, 2018)

PM 101 - FOUNDATION OF MATHEMATICS

(REPEAT)

Answer all questions   Time : Three hours

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1. (a) Let p and q be two statements such that $p \to \sim q$ is false. Find the truth value of each of the following statements:

   i. $p \wedge (q \to \sim p)$;

   ii. $q \wedge (p \vee \sim q)$.

   (b) Prove the following equivalences using the laws of logic:

   i. $(\sim p \wedge q) \vee p \equiv p \vee q$;

   ii. $[p \vee (q \wedge r)] \vee \sim [(\sim q \wedge \sim r) \vee r] \equiv p \vee q$,

   where p, q and r are statements.

   (c) Using the valid argument forms, deduce the conclusion t from the premises given below:

$$p \vee q$$

$$q \to r$$

$$p \wedge s \to t$$

$$\sim r$$

$$\sim q \to u \wedge s,$$

   where p, q, r, s, t and u are statements.

2. (a) Simplify the expression $[(A \cup \Phi) \cap (B \cup A') \cap (A \cup B \cup X)]'$ using the laws o

where A and B are subsets of a universal set X.

(b) For any sets A and B, prove that $A \triangle B = (A \cup B) \setminus (A \cap B)$.

Hence show that:

i. $A \triangle B$ and $A \cap B$ are disjoint;

ii. $A \cup B = (A \triangle B) \cup (A \cap B)$.

(c) Prove the following:

i. $(A \times C) \cup (B \times C) = (A \cup B) \times C$;

ii. $(A \setminus B) \times C = (A \times C) \setminus (B \times C)$.

3. (a) Let $S = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x \neq 0, y \neq 0\}$ and define a relation R o

$(x_1, y_1)R(x_2, y_2)$ if and only if $x_1y_1(x_2^2 - y_2^2) = x_2y_2(x_1^2 - y_1^2)$.

i. Show that R is an equivalence relation;

ii. If (a, b) is a fixed element of S, show that

$(x, y)R(a, b)$ if and only if $\frac{a}{b} = \frac{x}{y}$ or $\frac{a}{b} = -\frac{y}{x}$.

(b) Let R be an equivalence relation on a set A. Prove the following:

i. $[a] \neq \Phi$ for all $a \in A$,

ii. $aRb \iff [a] = [b]$,

iii. either $[a] = [b]$ or $[a] \cap [b] = \Phi$ for all $a \in A$.

4. (a) Define the following terms:

i. *injective mapping*, ii. *surjective mapping*, iii. *inverse mappin*

(b) The functions $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are defined by

$$f(x) = \begin{cases} 4x + 1, & \text{if } x \geq 0; \\ x, & \text{if } x < 0; \end{cases} \quad \text{and } g(x) = \begin{cases} 3x, & \text{if } x \geq 0; \\ x + 3, & \text{if } x < 0. \end{cases}$$

Show that $g \circ f$ is a bijection and give a formula for $(g \circ f)^{-1}$.

5. (a) Let $f : X \to Y$ be a mapping and A and B be any subsets of X. Prove the fol

i. f is injective if and only if $f(A \cap B) = f(A) \cap f(B)$;

ii. f is surjective if and only if $Y \setminus f(A) \subseteq f(X \setminus A)$.

(b) Show that the last element of every partially ordered set is a maximal eleme

Is the converse true? Justify your answer.

6. (a) State the *division algorithm*.

Show that the square of any odd integer is of the form $8k + 1$, where k is an integer.

(b) Using the Euclidean algorithm find integers x and y satisfying

$$gcd(341, 527) = 341x + 527y.$$

(c) 1000 glasses are packed in two types of boxes. There are 172 boxes in the first type and 20 in the second type. If each type contains a fixed number of glasses, find the number of glasses in each type.