EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

EXTERNAL DEGREE EXAMINATION IN SCIENCE ~~2008/2009~~ (2010/2011)

THIRD YEAR SECOND SEMESTER (Mar./May, 2016)

EXTMT 306 - PROBABILITY THEORY (REPEAT)

Answer All Questions **Time : Two Hours**

Calculators and Statistical tables will be provided

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1. (a) Define the "moment generating function" of a random variable X.

   Find the moment generating function for the Gamma distribution is given by

$$f_X(x) = \begin{cases} \dfrac{\lambda^m x^{m-1} e^{-\lambda x}}{\Gamma(m)} & \text{if} \quad x \geq 0 \\ 0 & \text{otherwise} \end{cases},$$

   Hence find the mean and variance.

   (b) Define a conditional probability.

   Let A and B be two events. Show that

$$P(A \mid B^C) = \frac{P(A) - P(A \cap B)}{1 - P(B)}.$$

   Hence show that $P(A \cap B) \geq P(A) + P(B) - 1$.

(c) If the waist measurements X of 800 buys are normally distributed with mean 66 c[m] and variance 25 cm, find the number of boys with waist greater than or equal to 7[...] cm.

2. (a) A continuous random variable X follows a normal distribution with

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, \qquad -\infty < x < \infty$$

where μ is the mean of X and σ is the standard deviation of X. Show that

i. $E(X) = \mu$;

ii. $Var(X) = \sigma^2$.

(b) The time taken by a milkman to deliver milk to the High street is normally di[s]tributed with mean 12minutes and standard deviation 2minutes. He delivers mi[lk] everyday. Estimate the number of days during the year when he takes

i. longer than 17 minutes;

ii. less than 10 minutes;

iii. between 9 and 12 minutes.

3. (a) If X is a random variable with probability density function f_X and g is monotonical[ly] increasing and differentiable function from $\mathbb{R}$ to $\mathbb{R}$. Show that $Y = g(X)$ has t[he] density function

$$f_Y(y) = f_X[g^{-1}(y)]\frac{d}{dy}[g^{-1}(y)], \quad y \in \mathbb{R}.$$

Let X be random variable with exponential distribution parameter λ. Find densi[ty] function of $2X + 5$.

(b) Random variables X and Y have joint density function

$$f_{XY} = \begin{cases} c(x^2 + \frac{1}{2}xy) & \text{if } 0 < x < 1; \\ 0 & \text{otherwise.} \end{cases}$$

Find

i. the value of c ;

ii. marginal density functions X and Y ;

iii. $E(XY)$.

4. (a) Define unbiased estimator.

Show that

$$\sum_{i=1}^{n}\left(\frac{X_i-\overline{X}}{\sigma}\right) \sim \chi^2_{n-1}$$

where $X_1, X_2, \cdots, X_n$ are identical random variables from the normal distribution with mean μ and variance σ^2.

Hence prove that S^2 is unbiased estimator for σ^2, where $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i-\overline{X})^2$.

(b) Let $X_1, X_2, X_3, ..., X_n$ be a random sample from an Exponential distribution with parameter λ. Show that $\frac{1}{\overline{X}}$ is the maximum likelihood estimator of parameter λ, where $\overline{X}$ is the sample mean.