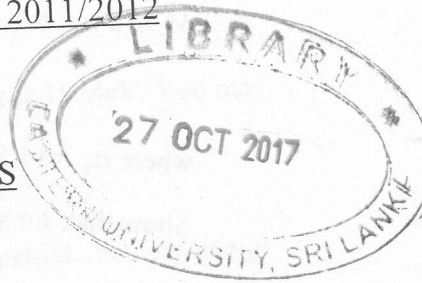


EASTERN UNIVERSITY, SRI LANKA

SPECIAL DEGREE EXAMINATION IN SCIENCE – 2011/2012

(SEPTEMBER/OCTOBER - 2016)

PH 406 ADVANCED NUCLEAR PHYSICS



Time: 02 hour

Answer ALL Questions

1. (a) What are the two kinds of experiments used to determine the radius of a nucleus?

(b) Assuming spherical shape for nucleus show that the radius of a nucleus with mass number A is $R = R_0 A^{1/3}$, where R_0 is a constant.

(c) The nuclear charge distribution will be described by the Fermi function of the form

$$\rho(r) = \frac{\rho_0}{(1 + e^{(r-R)/a})}$$

where the symbols have their usual meaning.

(i) what is the skin thickness?

(ii) Determine the value of the parameter a if the skin thickness is 2.3 fm.

(d) ^{25}Al and ^{25}Mg are mirror nuclei. ^{25}Al can decay into ^{25}Mg via β^+ - decay reaction, which has a Q-value of 3.26 MeV. Use the following given data to calculate the radius of ^{25}Al nucleus.

Mass of hydrogen atom, $m(^1\text{H}) = 938.79 \text{ MeV}/c^2$

Mass of neutron, $m_n = 939.57 \text{ MeV}/c^2$,

Mass of electron, $m_e = 0.511 \text{ MeV}/c^2$

Coulomb energy of a uniform charged sphere of radius R in fm is

$$E_c = \frac{3}{5} \frac{1}{4\pi\epsilon_0} \frac{Q^2}{R} = \frac{3}{5} \times \frac{1.44}{\epsilon^2} \times \frac{Q^2}{R}, \text{ in MeV}$$

02. (a) Consider the nucleons in $^{56}_{26}\text{Fe}$ and $^{238}_{92}\text{U}$. Show that the nucleons in Fe are more tightly bound than the nucleons in U.

(Nuclear masses of Fe = $5.2 \times 10^4 \text{ MeV}/c^2$ and U = $22.2 \times 10^4 \text{ MeV}/c^2$)

(b) The mass of an atom can be expressed on semi-empirical mass formula as

$$M(A, Z)c^2 = \left(Am_p c^2 - a_v A - a_s A^{\frac{2}{3}} - a_{sy} A \right) - ((m_n - m({}^1H))c^2 + 4a_{sy})Z + (a_c A^{\frac{-1}{3}} - 4A^{-1}a_{sy})Z^2$$

where $a_v = 15.5$ MeV, $a_c = 0.72$ MeV, $a_s = 16.8$ MeV and $a_{sy} = 23$ MeV.

Show that, for small A , atomic mass becomes minimum at $Z \sim \frac{A}{2}$.

(c) Explain why hyperfine splitting is small compared to fine structure splitting.

(d) (i) Draw an energy diagram for all possible fine structures and hyperfine structures of an electron in $n = 2$ state of the hydrogen atom.

(ii) What will happen to the above hyperfine structures under an external magnetic field?

Draw any changes for this situation in the above diagram.

03. Consider a finite 3-dimensional square potential well of width R and depth V_0 to describe the relative motion of the neutron and the proton of the deuteron as shown in the diagram.

$$V(r) = -V_0 \quad \text{for } 0 \leq r < R$$

$$= 0 \quad \text{for } R \leq r$$

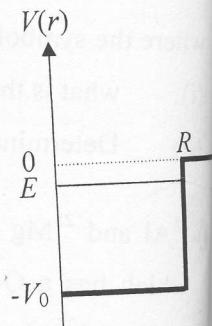
The Schrödinger equation for the relative motion of the neutron and the proton is given by

$$\left[-\frac{\hbar^2}{2\mu} \frac{d^2}{dr^2} + V(r) \right] U(r) = E U(r),$$

where the symbols have their usual meanings.

(a) Show that the relationship between R and V_0 is given by the transcendental equation

$$\cot kR = -\sqrt{\frac{B}{V_0 - B}}, \quad \text{where } k \text{ is given by } k = \sqrt{\frac{2\mu}{\hbar^2}(V_0 - B)} \quad \text{and } B \text{ is the energy of the deuteron.}$$



For $B \ll V_0$ show that the relationship between R and V_0 is given by $V_0 R^2 \approx \frac{\pi^2 \hbar^2}{4m}$,

where $m = m_p \approx m_n$.

(c) For $R = 2.1$ fm show that the depth of the potential well V_0 is about 35 MeV. You may assume that $\hbar c = 197$ MeV fm and $m = 939.6$ MeV/c².

4. According to the nuclear shell-model the ordering of the single particle nuclear energy levels is $1s_{1/2}$; $1p_{3/2}$; $1p_{1/2}$; $1d_{5/2}$; $2s_{1/2}$; $1d_{3/2}$; $1f_{7/2}$; $2p_{3/2}$

(a) Find the energy configuration of the protons and neutrons for the nucleus ${}^{23}_{11}\text{Na}$. Hence find the ground state spin and parity assignments of the nucleus. In this model the first excited states can be produced either

(i) by excitation of the unpaired nucleon into the next higher sub-shell, or

(ii) by pairing unpaired nucleon with another nucleon excited from the next lower sub-shell.

Determine the spin and parity for these two types of excited states for the above nuclide.

(b) According to shell model the observable magnetic moment of a nucleon in the state

(j, l) is given by $\mu = [g_l(j - \frac{1}{2}) + \frac{1}{2}g_s] \mu_N$ for $j = l + \frac{1}{2}$ and

$$\mu = \left[g_l \frac{j(j + \frac{1}{2})}{j + 1} + \frac{1}{2} \frac{1}{j + 1} g_s \right] \mu_N \quad \text{for } j = l - \frac{1}{2}.$$

Find the shell model configuration and the expected magnetic moment of ${}^{42}\text{Sc}_{21}$, if ${}^{42}\text{Sc}_{21}$ has a low lying level with spin and parity 7^+ .

For the proton $g_l = 1$ and $g_s = 5.5857$ and for the neutron $g_l = 0$ and $g_s = -3.8261$.