

EASTERN UNIVERSITY, SRI LANKA

## DEPARTMENT OF MATHEMATICS

THIRD YEAR EXAMINATION IN SCIENCE -2010/2011
FIRST SEMESTER-( April, 2013)
MT 305 - OPERATIONAL RESEARCH
Answer all questions.
Time: Three hours.

1. A company manufactures three types of bicycles $A, B$ and $C$. Each bicycle passes through three departments: Fabrication, Painting \& Plating, and Final Assembling. The relevant manufacturing data are given in the table as follows.

| Departments | Labor hours per Bicycle |  |  | Maximunn labor hours <br> available per day |
| :---: | :---: | :---: | :---: | :---: |
|  | A | B | C |  |
| Fabrication | 3 | 4 | 5 | 130 |
| Painting \& Plating | 5 | 3 | 5 | 120 |
| Final Assembling | 4 | 3 | 5 |  |
| Profit (in \$) per Bicycle | 80 | 100 | 70 |  |

(a) Build up a linear programming model for the problem of deciding how many of each type of bicycle to be produced to maximize the profit.
(b) Use the Simplex method to find the optimal solution for the above linear programming model.
(c) What is the maximum profit?
(d) Discuss the effect of the solution in part (b), when the profit on the type Cicycle increases to $\$ 110$ with all other data in part (a) remains the same.
02. Consider the following linear programming model.

Minimize

$$
\mathrm{Z}=7 \mathrm{X}_{1}+2 \mathrm{X}_{2},
$$

subject to the constraints:

$$
\begin{aligned}
& 2 \mathrm{X}_{1}+4 \mathrm{X}_{2} \geq 5, \\
& 8 \mathrm{X}_{1}+4 \mathrm{X}_{2} \geq 8, \\
& 3 \mathrm{X}_{1}+8 \mathrm{X}_{2} \geq 4, \\
& 3 \mathrm{X}_{1}-2 \mathrm{X}_{2} \geq 4,
\end{aligned}
$$

where $X_{1}, X_{2} \geq 0$.
(a) Discuss the possibility and advantages of applying Dual Simplex method for this model.
(b) Defining variables clearly, construct the dual of this primal.
(c) Find the solutions of the dual constructed in part (b).
(d) Interpret your solutions.
03. Using Revised Simplex method, solve the following linear programming model: Maximize

$$
Z=2 X_{1}+3 X_{2}-X_{3}+4 X_{4}
$$

subject to the constraints:

$$
\begin{aligned}
& X_{1}-2 X_{2}+X_{4} \leq 10 \\
& X_{1}+X_{2}+2 X_{3} \leq 16 \\
& (1 / 2) X_{2}-X_{3}-X_{4} \leq 8
\end{aligned}
$$

where $\mathrm{X}_{1}, \mathrm{X}_{2}, \mathrm{X}_{3}, \mathrm{X}_{4} \geq 0$.
04. A company has four machines $\mathrm{M}_{1}, \mathrm{M}_{2}, \mathrm{M}_{3}$ and $\mathrm{M}_{4}$ available for assignment to four tasks $T_{1}, T_{2}, T_{3}$ and $T_{4}$. Any machine can be assigned to any task, and each task requires processing by one machine. The time requires by each machine for processing of each task is given in the table below:

| Machines | Time (Hours) requires by tasks |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{T}_{\mathbf{1}}$ | $\mathbf{T}_{\mathbf{2}}$ | $\mathbf{T}_{\mathbf{3}}$ | $\mathbf{T}_{\mathbf{4}}$ |  |
| $\mathrm{M}_{1}$ | 13 | 4 | 7 | 6 |  |
| $\mathrm{M}_{2}$ | 1 | 11 | 5 | 4 |  |
| $\mathrm{M}_{3}$ | 6 | 7 | 2 | 8 |  |
| $\mathrm{M}_{4}$ | 1 | 3 | 4 | 9 |  |
|  |  |  |  |  |  |

(a) Formulate a mathematical model for this assignment problem. Clearly define the variables and state the constraints.
(b) Use Hungarian method to find the optimal assignments that minimize the total processing time.
(c) Write down the minimum total processing time.
05. A transporting company plans to transport some logs from three harvesting sites $\mathrm{S}_{1}, \mathrm{~S}_{2}$ and $S_{3}$ to three sawmills $M_{1}, M_{2}$ and $M_{3}$ at the minimum cost. The distance from each site to each sawmill, number of truckloads of logs available at each site and number of truckloads of logs each sawmill demands, are given in following table. The average cost of transportation is $\$ 2$ per kilometer for both loaded and empty trucks.

| Logging sites | Distance to mills(in km) |  |  | Maximum truckloads <br> from logging site per day |
| :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{M}_{1}$ | $\mathbf{M}_{\mathbf{2}}$ | $\mathbf{M}_{3}$ | 20 |
| $\mathrm{~S}_{1}$ | 8 | 15 | 50 | 30 |
| $\mathrm{~S}_{2}$ | 10 | 17 | 20 | 45 |
| $\mathrm{~S}_{3}$ | 30 | 26 | 15 | - |
| Mill demand <br> (Truckload per day) | 30 | 35 | 30 |  |

(a) Defining variables clearly, build up the mathematical model for the above transportation problem.
(b) Find the initial feasible solution by using Row minima method.
(c) Check the optimality of the solutions obtained in part (b) by using modified distribution (MODI) method.
(d) Find the minimum total cost.
06. Consider the road network as in the following figure, where distances (in km ) between adjacent cities are summarized. Find the shortest route from city 1 to city 10, by using Systematic method.


