EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

THIRD YEAR SPECIAL REPEAT EXAMINATION IN SCIENCE - 2010/2011

(JUNE / JULY, 2014)

MT 306 – PROBABILITY THEORY

Answer **all** questions | Time: **Two** hours

Statistical tables will be provided

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01. (a) State and prove the **Total Probability** theorem and **Bayes'** theorem.

(b) Three companies A, B and C produce same circuits used for computers. Contributions to the total production of each company are 40%, 25% and 35% respectively. Respective probabilities of circuits, produced by each company, being defective are, 20%, 25% and 30%. A circuit was selected at random and found to be a defective one. What is the probability that the selected circuit is from company B.

02. The diameter of certain oil seals manufactured by a company is normally distributed with mean of 20mm and standard deviation of 3mm. If an oil seal is selected randomly from this company, find the probability that diameter of the selected oil seal is:

(i) greater than 22mm;

(ii) less than 18mm;

(iii) between 17mm and 20mm.

(iv) How many oil seals will have more than 22mm of diameter, if 500 oil seals are taken from this company?

(v) What is the maximum diameter that 95% of the oil seals will have?

(P. T. O)

03. Continuous random variables X and Y have the following joint probability density function

$$f_{XY}(x,y) = \begin{cases} c\left(x^2 + \frac{1}{2}xy\right) & , \text{ if } 0 < x < 1, 0 < y < 2, \\ 0 & , \text{ otherwise}. \end{cases}$$

where c is a constant.

Find the followings:

(i) Value of c ;

(ii) The cumulative joint probability distribution function $F_{XY}(x,y)$;

(iii) Marginal density function of X and Y, $f_X(x)$ and $f_Y(y)$, respectively;

(iv) $E(X)$ and $E(Y)$.

04. (a) Assume $X_1, X_2, X_3, \dots\dots, X_n$ be a random sample from a Poisson distribution with parameter λ. Find an estimator for λ using method of moments. Use following data to estimate λ.
5, 9, 7, 8, 5, 5, 7, 6, 3, 8.

(b) Let $X_1, X_2, X_3, \dots\dots, X_n$ be a random sample from an Exponential distribution with parameter λ. Show that $\frac{1}{\overline{X}}$ is the maximum likelihood estimator for λ, where $\overline{X}$ is the sample mean.