EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

ERNAL DEGREE EXAMINATION IN SCIENCE 2008/2009

ECOND YEAR SECOND SEMESTER (Mar./May, 2016)

EXTMT 217 - MATHEMATICAL MODELING (REPEAT)

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all questions | Time : Two hours

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cribe the steps involved in a mathematical model building process.

e a mathematical formulation for the following problem:

k can be consumed either directly or in processed forms such as butter, yoghurt and ese etc. The company X is the sole producer of milk in a certain region and also owns factory to process milk into various forms. The problem for the company is to determine relative quantities of the various products necessary to achieve this goal.

plain the logistic model

$$\frac{dp}{dt} = ap - bp^2, \qquad p(t_0) = p_0,$$

he population growth of a single species.

d the population $p(t)$ and the limiting value of $p(t)$ for $t > t_0$.

sume that the global resources will provide enough food only for 6×10^{10} humans, the rld populations were 1.6×10^{10} and 2.4×10^{10} in the years 1900 and 1955, respectively. ng the logistic population model, predict the population for the year 2020.

3. Suppose a x force and a y force are engaged in combat. Let $x(t)$ and $y(t)$ respective strength of the forces at time t, when t is measured in days from the s combat. Conventional combat model is given by

$$\frac{dx(t)}{dt} = -a\,x(t) - b\,y(t) + P(t);$$
$$\frac{dy(t)}{dt} = -d\,y(t) - c\,x(t) + Q(t).$$

Explain the terms involved in these equations.

By using the assumptions that there is no reinforcement arrived and no operat occur, obtain a simplified model, and hence show that

$$x(t) = x_0 \cosh(\beta t) - \gamma y_0 \sinh(\beta t),$$

where $\beta = \sqrt{bc}$, $\gamma = \sqrt{\frac{b}{c}}$ and x_0, y_0 are the initial strength of the respective fo

4. The fish population in a certain part of the sea can be separated into prey pop fish) $x(t)$ and predator population(Selachians) $y(t)$. The model governing the in the selachians and food fish in the absence of fishing is given by

$$\frac{dx}{dt} = ax - bxy;$$

$$\frac{dy}{dt} = -cx + dxy.$$

Explain the terms involved in this model.

Show that $\frac{y^a}{e^{by}} \cdot \frac{x^c}{e^{dx}} = k$, where k is a constant.

Let $x(t)$ and $y(t)$ be the periodic solution of the above equations.

If $\overline{x} = \frac{1}{T}\int_0^T x(t)\,dt$ and $\overline{y} = \frac{1}{T}\int_0^T y(t)\,dt$ then show that $\overline{x} = \frac{c}{d}$ and $\overline{y} = \frac{a}{b}$ w period.

Hence show that a moderate amount of fishing increases the average numbe and decreases the average number of selachians.