

EASTERN UNIVERSITY, SRI LANKA DEPARTMENT OF MATHEMATICS

SECOND EXAMINATION IN SCIENCE - 2013/2014

## SECOND SEMESTER (Sep./Oct., 2016)

PM 205 - DIFFERENTIAL GEOMETRY

(Proper & Repeat)

all questions

Time : One hour

27 OCT 2017

Find the directions of the tangent, normal and binormal vectors and also find the osculating, normal and the rectifying planes at any point of the circular helix given by

$$\underline{r} = (a\cos u, a\sin u, bu).$$

) If  $\underline{r}_1$  is the position vector of a point  $P_1$  on the involute  $\tilde{C}$  of a curve C, then show that

$$\underline{r}_1 = \underline{r} + (\lambda - s)\underline{t}$$

where  $\underline{r}$  is the position vector of P on C,  $\underline{t}$  is the unit tangent at P on C,  $\lambda$  is an arbitrary constant and s is the arc-length parameter.

Find the involute of the cubic curve

$$\underline{r} = (3u, 3u^2, 2u^3).$$

a) Define the *curvature* and *torsion* at any point of a space curve.

- b) State and prove the Serret-Frenet formulas at a point P on a space curve  $\gamma$ .
- c) Using the Serret-Frenet formulas, find the curvature and torsion at an arbitrary point on the circular helix given by

$$\underline{r} = (a\cos\theta, a\sin\theta, a\theta\cot\alpha)$$

where a and  $\alpha$  are constants and  $\theta$  is a parameter.