EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

FIRST EXAMINATION IN SCIENCE - 2007/2008 & 2008/20

FIRST SEMESTER (Jun./Sept., 2015)

EXTERNAL DEGREE

EXTMT 101 - FOUNDATION OF MATHEMATICS

(REPEAT)

Answer all questions | Time : Three hours

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1. (a) Prove the following equivalences using the laws of algebra of logic:

   i. $p \vee (p \wedge q) \sim p$;

   ii. $(p \wedge q) \vee \sim p \equiv \sim p \vee q$;

   iii. $[p \vee (q \wedge r)] \vee \sim [(\sim q \wedge \sim r) \vee r] \equiv p \vee q$,

   where p, q and r are statements.

   (b) Test the validity of the argument "If you are a mathematician then you are clever. You are clever and rich. Therefore if you are rich then you are a mathematician".

2. (a) For any sets A and B, prove that $A \triangle B = (A \cup B) \setminus (A \cap B)$.

   Hence show that:

   i. $A \triangle B$ and $A \cap B$ are disjoint,

   ii. $A \cup B = (A \triangle B) \cup (A \cap B)$.

   (b) For any sets A, B and C, prove that:

   i. $A \times (B \cap C) = (A \times B) \cap (A \times C)$,

   ii. $A \times (B \setminus C) = (A \times B) \setminus (A \times C)$.

3. (a) Let ρ be a relation defined on $\mathbb{R}$ by $x\rho y \Leftrightarrow x^2 - y^2 = 2(y - x)$, where $\mathbb{R}$ de set of all real numbers.

   i. Prove that ρ is an equivalence relation.

   ii. Determine the $\rho-$class of 1.

   (b) Let R be an equivalence relation on a set A. Prove the following:

   i. $[a] \neq \Phi \quad \forall a \in A$,

   ii. $aRb \Leftrightarrow [a] = [b]$,

   iii. either $[a] = [b]$ or $[a] \cap [b] = \Phi \quad \forall a \in A$.

4. (a) Define the following terms:

   i. *injective mapping*,

   ii. *surjective mapping*,

   iii. *inverse mapping*.

   (b) The functions $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are defined by

$$f(x) = \begin{cases} 4x + 1, & \text{if } x \geq 0; \\ x, & \text{if } x < 0; \end{cases} \quad \text{and } g(x) = \begin{cases} 3x, & \text{if } x \geq 0; \\ x + 3, & \text{if } x < 0. \end{cases}$$

   Show that $g \circ f$ is a bijection and give a formula for $(g \circ f)^{-1}$.

5. (a) Let $f : X \to Y$ be a mapping. Prove that f is surjective iff $Y \backslash f(A) \subseteq f($ all subset A of X.

   (b) i. Prove that every partially ordered set has at most one last element.

   ii. Show that last element of every partially ordered set is a maximal ele

   Is the converse true? Justify your answer.

6. (a) State *division algorithm*.

   Show that the square of any odd integer is of the form $8k + 1$, where k is a

   (b) Using the Euclidean algorithm find the $gcd(341, 527)$.

   (c) A certain number of sixes and nines are added to give the sum of 126; if th of sixes and nines are interchanged, the new sis 114. How many of each originally?