EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

TERNAL DEGREE EXAMINATION IN SCIENCE - 2007/2008, 2008/2009

FIRST YEAR FIRST SEMESTER (Jun./Sep., 2015)

EXTMT 106 - TENSOR CALCULUS

(Repeat)

ver all questions — Time : One hour

(a) Define the following terms:

i. Covariant tensor,

ii. Contravariant tensor.

(b) Write the transformation equation for the following tensors:

i. A_k^{pt};

ii. B_{tk}^{p};

iii. D_{ptk}^{abc}.

(c) Let A_{rst}^{pq} be a tensor. Choose $p = t$ and show that A_{rsp}^{pq}, where the summation convention is employed, is a tensor. What is its rank ?

(d) The covariant components of a tensor in rectangular co-ordinate system are, xy, $2y - z^2$, xz. Find its covariant components in Spherical co-ordinate (r, θ, ϕ).

2. (a) Define the Christoffel's symbols of the first and second kind.

(b) Determine the Christoffel's symbols of the second kind for the metric

$$ds^2 = d\rho^2 + \rho^2 d\phi^2 + dz^2.$$

(c) With the usual notation, prove the following:

i. $\dfrac{\partial g_{pq}}{\partial x^m} = [pm, q] + [qm, p]$;

ii. $\dfrac{\partial g^{pq}}{\partial x^m} = -g^{pn}\Gamma^q_{mn} - g^{qn}\Gamma^p_{mn}$;

iii. $\dfrac{1}{2g}\dfrac{\partial g}{\partial x^m} = \Gamma^j_{jm}$.