

23 AUG 2013  
EASTERN UNIVERSITY, SRI LANKA

**EASTERN UNIVERSITY, SRI LANKA**  
**FIRST YEAR EXAMINATION IN ARTS (2011/2012)**  
**JULY, 2013**  
**CC101 - BASIC MATHS**  
**(EXTERNAL DEGREE)**

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**Answer five questions only**  
**Time: Three hours**

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1. (a) Simplify each of the following:

i  $\left(\frac{27b^3}{216}\right)^{\frac{1}{3}} \times \left(\frac{81a^2}{16b^2}\right)^{-\frac{1}{2}};$

ii  $\frac{5x^{-1}y^{-4}}{(3y^5)^{-2}x^9};$

iii  $\left(\frac{x^2 - xy}{xy + y^2} \div \frac{x^2 - y^2}{x^2 + 2xy + y^2}\right) \div \left(\frac{x^2 - 2xy + y^2}{x^2y - xy^2}\right).$

(b) Solve the following equation

$$2(2^{2x}) - 5(2^x) + 2 = 0.$$

(c) If  $a^2 + b^2 = 1$  and  $ab = \frac{1}{3}$  then prove that  $2 \log\left[\frac{a-b}{3}\right] = \log a + \log b$ .

2. (a) Factorize the following:

i  $16x^4 - 81y^4;$

ii  $a^2x^2 - 4ax - 21;$

iii  $6x^2 - 11xy + 3y^2.$

(b) If  $y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ , then find "a".

(c) If  $p = q^{2a}$ ,  $q = r^{2b}$  and  $r = p^{2c}$ , then prove that  $abc = \frac{1}{8}$ .

(d) Simplify the following equation

$$\log_3 243 + 2 \log_3 30 - (\log_3 100 + 3 \log_3 3).$$

3. (a) Solve the following simultaneous equations:

$$\frac{2}{x} + \frac{5}{y} = 5,$$

$$\frac{1}{x} - \frac{7}{y} = 12.$$

(b) Let  $\alpha_1$  and  $\alpha_2$  be the roots of the equation  $ax^2 + bx + c = 0$ , then prove that  $\alpha_1 + \alpha_2 = -b/a$  and  $\alpha_1\alpha_2 = c/a$ , where  $a \neq 0$ .

(c) Let  $\alpha, \beta$  be the roots of the equation  $ax^2 + bx + c = 0$ . Then find the quadratic equation with the solutions  $\alpha/\beta, \beta/\alpha$ .

4. (a) Write the rank of the following matrices:

i.  $\begin{pmatrix} 6 & -1 & 0 \\ 5 & 0 & 3 \\ 1 & 2 & 1 \\ 0 & 2 & 8 \\ 1 & 0 & 2 \end{pmatrix};$

ii.  $\begin{pmatrix} 5 & 0 & 6 \\ 1 & 4 & 0 \end{pmatrix};$

iii.  $(6 \ 1 \ 0 \ 5).$

(b) Let  $A = \begin{pmatrix} 4 & 5 \\ 1 & 0 \end{pmatrix}$  and  $B = \begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix}$ , then find  $3A - 5B - 2I$ , where  $I$  is a  $2 \times 2$  identity matrix.

(c) Let  $A = \begin{pmatrix} -1 & -1 & 3 \\ -2 & 2 & -2 \\ 0 & 1 & -1 \end{pmatrix}$ ,  $B = \begin{pmatrix} -1 & 0 & 3 \\ -2 & -2 & -1 \\ -3 & 1 & -1 \end{pmatrix}$  and  $C = \begin{pmatrix} 1 & -1 & 1 \\ -1 & 2 & 1 \\ -2 & 3 & 0 \\ 0 & 2 & 2 \end{pmatrix}$ , then find

i.  $A+B$ ;

ii.  $CB$ ;

iii.  $BA$ .



5. (a) Find the limit value of the following:

i.  $\lim_{x \rightarrow 2} \frac{x^2 - 4x + 4}{x - 2}$ ;

ii.  $\lim_{x \rightarrow \infty} \left( \frac{4x^4 + 5x^3 + 3}{2x^4 + 3x} \right)$ ;

iii.  $\lim_{x \rightarrow -2} \left( \frac{x^3 + 8}{x + 2} \right)$ .

(b) Differentiate the following with respect to  $x$  :

i  $y = \frac{x^2 + 1}{x - 1}$ ;

ii  $y = x^2(\ln x)^3$ ;

(c) Examine the maximum and minimum value of the function,  $f(x) = 2x^2 - 6x + 3 = 0$ .

6. (a) Integrate the following with respect to  $x$  :

i.  $\int x^4(1+x^5)^{1/3} dx$ ;

ii.  $\int \frac{2(x+1)}{x^2 + 2x + 7} dx$ ;

iii.  $\int \frac{e^x}{1+e^x} dx$ .

(b) Evaluate the following:

i.  $\int_1^4 (2x^{-3} + 4x^{1/2}) dx$ ;

ii.  $\int_0^2 \frac{x^2}{\sqrt{1+3x^3}} dx$ .

7. (a). Let  $U$  be a given set and  $P$  and  $Q$  are subsets of  $U$ . If  $n(U) = 700$   
 $n(P) = 200$ ,  $n(Q) = 300$ ,  $n(P \cap Q) = 100$ , then find  $n[(P \cup Q)^c]$ .

(b). As per the conversation with 80 students in a school, 36 students follow Mathematics, 42 students follow Science, 30 students follow Tamil, 17 students follow Tamil and Science, 14 students follow Tamil and Mathematics, 8 students follow all three subjects. 15 students do not follow these three subjects. Draw a suitable Venn diagram for the above data. Using the Venn diagram, find out

- i. Those who are following Mathematics and Science but not Tamil.
- ii. Those who are following only Mathematics.
- iii. How many students follow only two subjects?
- iv. How many students do not follow Tamil among those who are following Mathematics?

8. (a). There are 10 balls in a paper bag with same dimensions; they are 5 white balls, 3 red balls and 2 black balls. A ball is selected randomly out, and then another ball is taken out from the bag without putting the first ball in it. Draw the sample space in a coordinate plane. Find the probability that the taken two balls are

- i. white colour;
- ii. red colour;
- iii. black colour;
- iv. one is white and other one is red;
- v. different colours.

(b).

- i. Find the equation of the straight line which is perpendicular to the line  $y = 3x + 2$  and passes through the point  $(1, 2)$ .
- ii. Find the equation of the straight line which passes through the points  $A = (3, 4)$ ,  $B = (4, 5)$ .



9. When the suitable persons are selected from a competitive exam to follow a course, the marks distribution obtained by all candidates are given below.

Class interval of Marks	Number of Examiners
11 - 20	3
21 - 30	6
31 - 40	22
41 - 50	54
51 - 60	49
61 - 70	31
71 - 80	18
81 - 90	11
91 - 100	6

- i. What is the modal class and median class of the distribution?
- ii. If the center value of the median class is the assumed mean, then find the true mean value.