EASTERN UNIVERSITY, SRI LANKA

EXTERNAL DEGREE EXAMINATION IN SCIENCE - 2009/201

SECOND YEAR FIRST SEMESTER (June / Sept. , 2012)

EXTMT 203 - EIGENSPACE AND QUADRATIC FORMS

(PROPER & REPEAT)

Answer all Questions | Time: Two hours

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1. (a) Define the term eigenvalue and eigenvector of a linear transformation.

   Find the eigenvalues and eigenvectors of the matrix

$$\begin{pmatrix} 1 & -1 & 1 \\ -1 & 1 & 1 \\ -1 & -1 & 3 \end{pmatrix}.$$

   (b) i. Prove that eigenvectors that corresponding to distinct eigenvalues of a linear transformation $T : V \to V$ are linearly independent.

   ii. Show that 0 is an eigenvalue of T if and only if T is singular.

   iii. Suppose λ is an eigenvalue of an invertible operator T. Show that λ^{-1} is an eigenvalue of T^{-1}.

   (c) Orthogonally diagonalize the matrix

$$A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}.$$

2. Define the term minimum polynomial of a square matrix.

   (a) State the Cayley - Hamilton theorem.

   Find the minimum polynomial of the square matrix

$$\begin{pmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -2 & 4 \end{pmatrix}.$$

   (b) Prove that for any square matrix A, the minimum polynomial exists and unique.

   (c) Let $M = \begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix}$, where A and B are square matrices. Show that th minimum polynomial $m(t)$ of M is the least common multiple of the minimu polynomials $g(t)$ and $h(t)$ of A and B respectively.

3. (a) Find an orthogonal transformation which reduces the following quadratic for to a diagonal form

$$5x_1^2 + 6x_2^2 + 7x_3^2 - 4x_1x_2 + 4x_2x_3 = 1.$$

   (b) Simultaneously diagonalize the following pair of quadratic forms

$$\phi_1 = x_1^2 - x_2^2 - 2x_3^2 - 2x_1x_2 + 4x_2x_3,$$

$$\phi_2 = x_1^2 + 2x_2^2 + 2x_3^2 - 2x_1x_2 - 2x_2x_3.$$

4. (a) What is meant by an inner product on a vector space.

   Let $x = (x_1, x_2, ..., x_n)$, $y = (y_1, y_2, ..., y_n) \in \mathbb{R}^n$, where $x_i, y_i \in \mathbb{R}$, $i = 1, 2, ...,$

   Let the inner product $< ., . >$ be defined on $\mathbb{R}^n$ as

$$< x, y >= xy^T = \sum_{i=1}^{n} x_i y_i.$$

   Show that $(\mathbb{R}^n, < ., . >)$ is an inner product space.

   (b) State and prove Cauchy Schwarz Inequality.

   (c) State the Gram Schmidt Process.

   Find the orthonormal set for span of M in $\mathbb{R}^4$, where

$$M = \{(1, 0, -1, 0)^T, (0, 1, 2, 1)^T, (2, 1, -1, 0)^T\}.$$