EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

EXTERNAL DEGREE THIRD EXAMINATION IN SCIENCE -2009/2010

FIRST SEMESTER- (December, 2012 / January, 2013)

EXTMT 306– PROBABILITY THEORY

ıswer **all** questions. Time: **Two** hours.

ıtistical tables will be provided.

. (a) State and prove the **Total Probability** theorem and **Bayes'** theorem.

(b) Four factories (A, B, C, D) of the same company manufacture tires. Productions of the each factory are 40%, 30%, 20% and 10% of total number of tires, respectively. The percentages of defective output of these factories are respectively 2%, 3%, 5% and 4%. A tire was selected at random and found to be defective. What is the probability that the selected tire was produced by factory C?

. (a) In a shooting game, shooter can get a gift if shooter can shoot at a target at least 3 times out of 5 trials. The probability that a shooter being successful at a given trial is 0.7. Find the probability that the shooter will get a gift.

(b) Students in a primary school are being tested to see how good their motor skills are. They need an idea of the dexterity of the students before ordering some new equipments. For this, a standard dexterity test which is normally distributed with population mean 10 and standard deviation 2.5 marks, respectively, is to be used. Find out

(i) the probability that an individual randomly selected, will take more than 15 marks;

(ii) how many students will score less than 15 marks, when 200 students take the same dexterity test;

(iii) 95% of the students will score more than what number of marks.

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(P. T. O.)

03. Continuous random variables X and Y have the following joint probability density functio

$$f_{X,Y}(x, y) = \begin{cases} e^{k(x+y)} & ; \ x \geq 0, y \geq 0 \\ 0 & ; \ otherwise \end{cases}$$

Find the following:

(i) Value of k ;

(ii) The cumulative joint probability distribution function $F_{X,Y}(x,y)$;

(iii) $E(XY)$;

(iv) Marginal density function of X and Y, $f_X(x)$ and $f_Y(y)$ respectively ;

(v) $E(X)$ and $E(Y)$.

04. (a) Assume $X_1, X_2, X_3, \ldots, X_n$ be a random sample from a Poisson distribution with paramete

(i) Find an estimator for λ using method of moment.

(ii) Estimate λ using given sample data (Sample data: 5, 9, 7, 8, 5, 5, 7, 6, 3, 8).

(iii) Check the unbiasedness of derived estimator in part (i).

(b) Let, $X_1, X_2, X_3, \ldots, X_n$ be a random sample from an Exponential distribution

parameter λ. Show that $\frac{1}{\overline{X}}$ is the maximum likelihood estimator of parameter λ, w

$\overline{X}$ is the sample mean.

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