

EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

FIRST EXAMINATION IN SCIENCE - 2011/2012

(EXTERNAL DEGREE)

SECOND SEMESTER (July/Aug., 2015)

MT 105 - THEORY OF SERIES

(REPEAT)



Answer all questions

Time: One hour

1. Define what is meant by the convergent and divergent of an infinite series of real numbers $\sum_{k=1}^{\infty} a_k$.

(a) A necessary condition for a series $\sum_{n=1}^{\infty} a_n$ to converge is that $\lim_{n \rightarrow \infty} a_n = 0$.

Is it true that, it is a sufficient condition for the convergence of the series $\sum_{n=1}^{\infty} a_n$? Justify your answer.

Does the following series converge or diverge? Explain your answer.

$$\sum_{n=1}^{\infty} 2n \sin\left(\frac{1}{n}\right)$$

[50 marks]

(b) Prove that the geometric series

$$\sum_{n=1}^{\infty} ar^{n-1}$$

converges if $|r| > 1$ and diverges otherwise. Where a and r are real constants.

Does the following series converge or diverge? If it converge, find the sum.

If it diverge, explain why?

$$\sum_{n=1}^{\infty} \left(\frac{2^n + 3^n}{4^n} \right).$$

[50 marks]

2. (a) i. Check the convergence of the following series by using the limit form of comparison test

$$\sum_{n=1}^{\infty} \frac{1}{n^2}.$$

[25 marks]

- ii. Use the root test to determine whether the series

$$\sum_{n=1}^{\infty} \frac{1}{[\ln(n+1)]^n}.$$

converges or diverges.

[20 marks]

- iii. Investigate whether the following series is convergent or divergent by using the alternating series test

$$\sum_{n=2}^{\infty} \frac{\cos(n\pi)}{\sqrt{n}}.$$

[25 marks]

- (b) Find the radius of convergence and interval of convergence of the following power series.

$$\sum_{n=1}^{\infty} \frac{(2x-5)^n}{n^2 3^n}.$$

[30 marks]