ver all questions   Time : Two hours

(a)   i. Define the function $f(x) = \dfrac{x^2 + x + 2}{x + 1}$ in Mathematica.

ii. Evaluate $f(10)$.

iii. Evaluate $f(x)$, for $x = -3, \ -2, \ -1, \ 0, \ 1, \ 2, \ 3$.

(b)   i. Factor $x^3 - 2x^2 + 3x$.

ii. Expand $(x + y - 2)(2x - 3)$.

iii. Simplify the given expression $\dfrac{\left(\frac{2}{x} - 3\right)}{1 - \frac{1}{x-1}}$.

(c)   i. Evaluate $\displaystyle\int \frac{x^5 + x^2 + x + 2}{(x^2 + 1)^2}\, dx$.

ii. Evaluate $\displaystyle\lim_{x \to 0} \left(\frac{\sin(4x)}{\tan x} \right)$.

iii. Compute the first eight derivatives of $f(x) = \sin x$. What is the 255th derivative of f ?

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2. (a) i. Find an equation of the line tangent to the graph of $f(x) = 9 - 4x^2$ at the point $(1, f(1))$ and sketch the graph of the tangent line.

   ii. Determine whether the two curves $y = x + 1$ and $y = x^2 - 1$ intersect each other, and if so, at how many intersecting points. What are the coordinates of those intersecting points?

   iii. Plot a graph showing the region under the curve $y = x^4$ from $x = -1$ to $x = 2$, and then find area of the region.

   iv. Sketch and identify the curve given by the parametric equations $x = t^2 - 2t$, $y = t + 1$ for $0 \le t \le 4$.

   (b) i. Let $A = \begin{pmatrix} 3 & -2 & 2 & 1 \\ 2 & 7 & -3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 \\ 3 & 2 \\ -4 & -3 \\ 0 & -2 \end{pmatrix}$. Find A^T, B^T and verify that $(AB)^T = B^T A^T$.

   ii. Find all critical numbers for the function $f(x) = x^{4/5}(x - 4)^2$.

   iii. Find $r(t)$ and $v(t)$ if $a(t) = ti + 4j$ subject to the initial conditions $v(0) = 3i - 2j$ and $r(0) = 0$.

3. (a) The temperature in an oven is 350° F when the oven is turned off. After 15 minutes, the temperature is 250° F. Assume the temperature in the house is 70° F.

   i. Use mathematica and Newton's Law of Cooling to find the temperature of the oven at any time t.

   ii. At what time the temperature will become 75° F?

   iii. What will be the temperature, in the limit as $t \longrightarrow \infty$?

(b) Let S_n be the nth partial sum of the harmonic series

$$\sum_{k=1}^{\infty} \frac{1}{k}.$$

i. Find S_{100}.

ii. Compute every 1000th partial sum up to $n = 10,000$.

iii. Plot the graph of the partial sums.