EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

SECOND EXAMINATION IN SCIENCE - 2012/2013

FIRST SEMESTER (Feb./Mar., 2016)

AM 207 - NUMERICAL ANALYSIS

(Proper & Repeat)

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er all Questions   Time: Two hours

(a) Define what is meant by:

i. *absolute error*;

ii. *relative error* .

Let $p = 0.54617$ and $q = 0.54601$. Use four-digit arithmetic to approximate $p - q$, and determine the absolute and relative errors when rounding and chopping.

(b) i. Determine the Taylor polynomial $P_4(x)$ of degree 4 for the function e^x around $x_0 = 0$.

ii. Write the formula for the remainder term $R_4(x) = e^x - P_4(x)$ as determined by Taylor's Theorem.

iii. Use the formula for the remainder term in part (ii) to estimate the error in approximating $e^{0.5}$ by $P_4(0.5)$.

iv. Determine the actual absolute error for the estimate $P_4(0.5)$ of $e^{0.5}$.

2. (a) Let $x = \phi(x)$ be the rearrangement of the equation $f(x) = 0$ and define iteration,

$$x_{n+1} = \phi(x_n), \qquad n = 0, 1, \ldots..$$

with the initial value x_0. If $\phi'(x)$ exists and is continuous such that $|\phi'(x)| \leq K$ for all x, then show that the sequence x_n generated by the above iteration conve to the unique root α of the equation $f(x) = 0$.

The function $g(x) = x^3 - x^2 - 4x + 5$ has three fixed points. One of them is x

i. Find the other two fixed points.

ii. Does fixed point iteration of g converge for x_0 near 1? Explain.

(b) i. Obtain Newton Raphson method to compute the root of the above equa in an interval $[a, b]$. Then use it to approximate the solution to $x + e^x$ with an error of at most 10^{-4}.

ii. Define the order and the asymptotic error constant of the iteration met to compute the non linear equation

$$f(x) = 0.$$

Hence show that the asymptotic error constant of the Newton Raphson met is $\frac{1}{2}\frac{f''(\alpha)}{f'(\alpha)}$.

3. (a) Define the divided differences $f[x_i, x_{i+1}, \ldots, x_{i+k}]$ for a function $f(x)$.

(b) Consider the quadratic polynomial

$$p_2(x) = f[x_0] + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1).$$

Show that this polynomial interpolates $f(x)$ at the points $(x_i, f(x_i))$, $i = 0, 1,$

(c) Use divided differences to construct the quadratic polynomial $p_2(x)$ that pas through the points

$$(0.1, 0.1248), \ (0.2, 0.2562) \text{ and } (0.4, 0.6108).$$

(d) Given that all these points lie on the curve $y = f(x)$, use the polynomial $p_2(x)$ the previous part to estimate $f(0.3)$.

) With the usual notations, the Trapezoidal rule is given by

$$\int_{x_i}^{x_{i+1}} f(x)dx = \frac{h}{2}(f_i + f_{i+1}) - \frac{1}{12}h^3 f''(\xi_i), \text{ where } \xi_i \in [x_i, x_{i+1}].$$

Obtain the composite Trapezoidal rule and derive a formula for the error.

i. Evaluate the integral $\int_0^{1/10} \frac{dx}{x+1}$ using the composite trapezoidal rule with 5 steps (subintervals).

ii. Use the error formula to estimate the error in part (i).

) The upward velocity of a rock is given at three different times in the following table

Time, t(s)	Velocity, v(m/s)
5	106.8
8	177.2
12	279.2

The velocity data is approximated by a polynomial as

$$v(t) = a_1 t^2 + a_2 t + a_3, \qquad 5 \le t \le 12$$

Find the values of a_1, a_2 and a_3 using the Gauss-Siedel method. Assume an initial guess of the solution as

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$$

and conduct two iterations.