EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

ECOND EXAMINATION IN SCIENCE - 2014/2015

FIRST SEMESTER (Nov./Dec., 2016)

AM 207 - NUMERICAL ANALYSIS

(Proper)

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ll Questions | **Time: Two hours**

Define what is meant by:

i. *absolute error*;

ii. *relative error* .

Use three digit rounding arithmetic to compute $\dfrac{\frac{13}{14}-\frac{6}{7}}{2e-5.4}$ and determine the absolute and relative errors.

i. Find the third Taylor polynomial $P_3(x)$ for the function $f(x) = (x-1)\ln x$ about $x_0 = 1$. Use $P_3(0.5)$ to approximate $f(0.5)$.

ii. Find an upper bound for the error $|f(0.5) - P_3(0.5)|$ using the error formula, and compare it to the actual error.

iii. Find a bound for the error $|f(x) - P_3(x)|$ in using $P_3(x)$ to approximate $f(x)$ on the interval $[0.5, 1.5]$.

2. (a) i. Let $x = \phi(x)$ be the rearrangement of the equation $f(x) = 0$ and define iteration, $x_{n+1} = \phi(x_n)$, $n = 0, 1, \ldots..$, with the initial value x_0. If $\phi'(x)$ ex and is continuous such that $|\phi'(x)| \leq K < 1$ for all x, then show that sequence x_n generated by the above iteration converges to the unique ro of the equation $f(x) = 0$.

ii. Find the fixed points of the function $f(x) = \frac{5}{2}x(1-x)$ by solving $f(x)$ = and the interval of convergence for the fixed point method such that wi contains one of this fixed point.

(b) i. Define the order and the asymptotic error constant of the iteration me to compute the non linear equation $f(x) = 0$.

ii. Obtain Newton Raphson method to compute the root of the above equa in an interval $[a, b]$.

Then use it to find the root of $f(x) = \ln(x) - \sin(x)$ with an initial gues $x_0 = 3$, accurate to $\epsilon = 10^{-2}$ in the function value.

3. (a) If $f \in C^{n+1}[a, b]$ and $P_n(x)$ is the Lagrange's interpolating polynomial wh interpolates the function $f(x)$ at the distinct points $x_0, x_1, \ldots, x_n$ in [prove that for all $\xi \in [a, b]$, there exist $\xi \in (a, b)$ such that

$$f(x) - P_n(x) = (x - x_0)(x - x_1)\ldots(x - x_n)\frac{f^{n+1}(\xi)}{(n+1)!}, \text{ where } \xi \in (a, b).$$

(b) i. Use Lagrange's method to find the interpolating polynomial for the data

i	0	1	2	3
x_i	0	1	2	3
$\sin x_i$	0	0.841	0.909	0.141

ii. Approximate $\sin(1.571)$ using the polynomial obtained in part (i).

iii. Find an upper bound on the error for the Lagrange interpolating polynom on the interval $[0, 3]$.

With the usual notations, the Simpson's rule is given by

$$\int_{x_{i-1}}^{x_{i+1}} f(x)dx = \frac{h}{3}\left(f_{i-1} + 4f_i + f_{i+1}\right) - \frac{1}{90}h^5 f^{(iv)}(\xi_i), \text{ where } \xi_i \in [x_i, x_{i+1}].$$

Obtain the composite Simpson's rule and show that the composite error is less than or equal to

$$\frac{1}{180}h^4(b-a)\left|f^{(iv)}(\xi)\right|, \qquad \text{where } \left|f^{(iv)}(\xi)\right| = \max_{a\le x\le b}\left|f^{(iv)}(x)\right|.$$

Determine the step size h required in order for the composite Simpson's rule to approximate the integral

$$\int_0^8 x \sin x$$

with an error of at most 10^{-4}.

Find the solution of the system of equations

$$\begin{aligned}
10x_1 - x_2 + 2x_3 &= 6,\\
-x_1 + 11x_2 - x_3 + 3x_4 &= 25,\\
2x_1 - x_2 + 10x_3 - x_4 &= -11,\\
3x_2 - x_3 + 8x_4 &= 15
\end{aligned}$$

correct to three decimal places, using the Gauss-Seidel iteration method.