EASTERN UNIVERSITY, SRI LANKA

THIRD EXAMINATION IN SCIENCE - 2012/2013

SECOND SEMESTER (June/July, 2016)

AM 310 - FLUID MECHANICS

SPECIAL REPEAT

er all questions | Time : Two hours

(a) Derive the continuity equation for a fluid flow in the form

$$\frac{D\rho}{Dt} + \rho \underline{\nabla} . \underline{V} = 0,$$

where ρ and $\underline{V}$ are the density and the velocity of the fluid.

Hence, establish the equation of continuity for an incompressible fluid in the form $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$ in cartesian coordinates, where u, v and w are the cartesian components of the velocity.

(b) Show that $\frac{k}{r^5}\left(3x^2 - r^2, 3xy, 3xz\right)$, where $r^2 = x^2 + y^2 + z^2$ and k is a constant, represents the velocity field in a possible fluid motion.

Show also that this motion is irrotational and hence determine the streamlines.

(a) With the usual notation, derive the *Euler's equation* for an incompressible and inviscid fluid flow.

Hence show that if the fluid flow is steady the *Euler's equation* can be written as

$$(\underline{V}.\underline{\nabla})\underline{V} = \underline{F} - \frac{1}{\rho}\underline{\nabla} p.$$

(b) An incompressible and inviscid fluid obeying Boyle's law $p = k\rho$, where k is a constant, is in motion in a uniform narrow tube. Prove that if ρ be the density of the fluid then the velocity v at a distance x at time t in the tube is given by the equation

$$\frac{\partial^2 \rho}{\partial t^2} = \frac{\partial^2}{\partial x^2}[(v^2+k)\rho].$$

(c) State the *Kelvin circulation theorem.*

If the velocity field is given by $\underline{V} = \dfrac{-y\underline{i} + x\underline{j}}{x^2+y^2}$ then calculate the circulation around a square with its corners at $(1,0),(2,0),(2,1),(11)$.

3. Let a gas occupy the region $r \leq R$, where R is a function of time t, and a liquid of constant density ρ lie outside the gas. By assuming that there is contact between the gas and the liquid all the time and that the motion is symmetric about the origin $r=0$, show that the motion is irrotational.

If the velocity at $r = R$, the gas liquid boundary is continuous then show that the pressure p at a point $P(\underline{r}, t)$ in the liquid is given by

$$\frac{p}{\rho} + \frac{1}{2}\left(\frac{R^2\dot{R}}{r^2}\right)^2 - \frac{1}{r}\frac{d}{dt}(R^2\dot{R}) = f(t), \text{ where } r = |\,\underline{r}\,|.$$

Further, if it is given that the liquid extends to infinity and is at rest with constant pressure Π at infinity, prove that the gas liquid interface pressure is equal to $\Pi + \dfrac{\rho}{2R^2}\dfrac{d}{dR}(R^3\dot{R}^2)$.

If the gas obeys the Boyle's law $pv^{4/3}$ = constant, where v is the volume of the gas, and expands from rest at $R = a$ to a position of rest $R = 2a$, show that the ratio of initial pressure of the gas to the pressure of the liquid at infinity is 14:3.

4. Write down the *Bernoulli's equation* for steady motion of an inviscid incompressible fluid.

A three-dimensional doublet of strength μ whose axis is in the direction of $\overrightarrow{Ox}$ is distant a from the rigid plane $x = 0$ which is the sole boundary of liquid of density ρ, infinite in extent. If the pressure at infinity is Π, find the pressure at a point on the boundary distant r from O.

Show that the pressure on the plane is least at a distant $\dfrac{\sqrt{5}a}{2}$ from the doublet.