EASTERN UNIVERSITY, SRI LANKA

SECOND EXAMINATION IN SCIENCE 1994/95 & 95/96

(August/September'97)

MT205 & 208 – MATHEMATICAL METHODS & NUMERICAL ANALYSIS

Time: Two hours.

Answer only four questions selecting two from each section.

SECTION A

MATHEMATICAL METHODS

1. (a) i. Write the transformation equations for the following tensors.

   A^i_{jk}, B^{ijk}_l, C_{mn}.

   ii. If $\overline{A}^p_r = \frac{\partial \overline{x}^p}{\partial x^q} \frac{\partial x^s}{\partial \overline{x}^r} A^q_s$, then prove that $A^q_s = \frac{\partial x^q}{\partial \overline{x}^p} \frac{\partial \overline{x}^r}{\partial x^s} \overline{A}^p_r$.

   (b) If $A(p, q, r) B^{qs}_r = C^s_p$, where B^{qs}_r is an arbitrary tensor and C^s_p a tensor, then prove that $A(p, q, r)$ is a tensor.

   (c) Find the covariant and contravariant components of a tensor in cylindrical co-ordinates (ρ, ϕ, z) if its covariant components in rectangular co-ordinates are $xy, 2y - z^2, xz$.

2. (a) Define the following:

   i. Christoffel symbols of first and second kind.

   ii. Geodesics.

(b) Prove the following:

i. $[pq, r] = [qp, r]$.

ii. $\Gamma^s_{pq} = \Gamma^s_{qp}$.

iii. $[pq, r] = g_{rs}\,\Gamma^s_{pq}$.

(c) Find the second kind of the Christoffel symbol and Corresponding Geodesic Eq in spherical co-ordinates (r, θ, ϕ).

3. (a) i. Explain the term **"Covariant derivative"** as applied to a tensor of type A^a_{bc}.

ii. Write the covariant derivative with respect to x^q of each of the following tensors.

$$A^{jk}_{l}, \quad A^{j}_{klm}, \quad A^{jk}_{lmn}.$$

(b) With the usual notations, prove the following:

i. $\dfrac{\partial g_{pq}}{\partial x^m} = [pm, q] + [qm, p]$;

ii. $\dfrac{\partial g^{pq}}{\partial x^m} = -\,g^{pn}\,\Gamma^q_{mn} - g^{qn}\,\Gamma^p_{mn}$.

Deduce that the covariant derivatives of the tensors g_{jk}, g^{jk}, δ^j_k are zero.

(c) Using the covariant derivative of metric tensor, prove that

$$\Gamma^e_{ca} = \frac{1}{2}\, g^{eb}\left[\partial_c(g_{ab}) + \partial_a(g_{cb}) - \partial_b(g_{ca})\right],$$

where $\partial_i = \dfrac{\partial}{\partial x^i}$.

SECTION B.

NUMERICAL ANALYSIS

4. (a) Use induction on r to show that the Horner algorithm

$$\begin{aligned} k_0 &= a_n, \\ k_r &= b\, k_{r-1} + a_{n-1} \text{ for } r = 1, 2, \ldots, n, \end{aligned}$$

generates a sequence (k_r) that satisfies

$$k_r = a_n\, b^r + a_{n-1} b^{r-1} + \cdots + a_{n-r} \text{ for } r = 0, 1, \ldots n.$$

Show also that

$$a_n\, x^n + a_{n-1}\, x^{n-1} + \ldots + a_0 = (x - b)(k_0\, x^{n-1} + k_1\, x^{n-2} + \cdots + k_{n-1}) + k_n.$$

Hence find the following:

i. the quotient polynomial and remainder when $p(x) = 3x^5 + 5x^4 + 8x^2 + 7x + 4$ is devided by $x + 2$;

ii. the Taylor series of $p(x)$ about the point $x = -2$.

(b) Explain what is meant by

i. fixed point representation ,

ii. floating point representation.

For base 16, round the number $\dfrac{\pi}{2} = (1.921FB54\ldots)_{16}$ to

i. 5 places in fixed point ,

ii. 5 digits in floating point.

(In base 16, $A = 10, B = 11, C = 12, D = 13, E = 14, F = 15$.)

5. (a) Define the term "rate of convergence" of direct iteration used to find the roots of any non-linear equation.

(b) Let $f : [a, b] \longrightarrow \Re$ be a continuous and differentiable function on $[a, b]$. State the Newton-Raphson iteration to find the roots of the equation $f(x) = 0$.

(c) Show that the order of convergence of the Newton-Raphson method is two.

(d) Use the Newton-Raphson method to find the real positive root of the equation $f(x) = x^3 - x^2 - x - 3 = 0$ accurate to two decimal places.
[Hint: Estimate $f(2)$ and $f(3)$]

(e) Which cases will the Newton-Raphson method fail?

6. (a) Let $f : [a, b] \longrightarrow \Re$ and let $x_0, x_1, \ldots, x_n$ be distinct points in $[a, b]$. Define the fundamental Lagrange polynomials $l_0, l_1, l_2, \ldots, l_n$ for these interpolation points and show that the polynomial P given by

$$P(x) = \sum_{j=0}^{n} f(x_j) l_j(x),$$

the Lagrange intrpolation polynomial, interpolates f at $x_0, x_1, x_2, \ldots, x_n$.

Show also that this is the unique polynomial of degree atmost n that interpolates f at these points.

Find the lagrange interpolation polynomial for $f(x) = \frac{1}{x}$ using the distinct points $x_0 = 2,\ x_1 = 2.5,\ x_2 = 4$.

(b) Use the Newton interpolation divided difference formula to approximate $f(0.05)$ from the following data.

x	0.0	0.2	0.4	0.6
f(x)	15.0	21.0	30.0	51.0

(c) Find the area, in the first quadrant, bounded by the curve $y = (x - 1)(x + 5)(9 - x)$ and x-axis by using composite Simson rule with step-size $h = 1$.