EASTERN UNIVERSITY, SRI LANKA

SECOND EXAMINATION IN SCIENCE (2001/2002)

FIRST SEMESTER

(April/May ' 2002)

MT 207 - NUMERICAL ANALYSIS

Answer all Questions Time : Two hours

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1. Define:

   - absolute error;
   - relative error.

   What is meant by saying that a number $\bar{x}$ approximates x to d significant digits?

   Illustrate, with an example, the loss of significance phenomenon.

   (a) Suppose x whose actual value is 2.0 is measured as 2.05.

      i. Give the relative error occurred in measuring x.

      ii. Compute x^2, x^3, x^4 and find the relative error in each computation.

      iii. Suggest a formula for the relative error in computing x^k for a positive integer k.

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(b) Compute:

i. $f(x) = x\left[\sqrt{x+1} - \sqrt{x}\right]$ at $x = 500$ to 6 significant digits accuracy;

ii. $f(x) = \dfrac{e^x - 1 - x}{x^2}$ at $x = 0.01$ to 6 significant digits accuracy;

iii. The roots of the quadratic equation

$$x^2 - 26x + 1 = 0$$

to 4 digit accuracy.

2. (a) Describe **Newton's method** to solve a system of non - linear equations

$$\begin{aligned} f(x,y) &= 0 \\ g(x,y) &= 0 \end{aligned}$$

Use the method to solve the system

$$\begin{aligned} x^2 - 2x - y + 0.5 &= 0 \\ x^2 + 4y^2 - 4 &= 0 \end{aligned}$$

with the starting values (2.00, 0.25)

(b) Describe an algorithm to find the zeros of a polynomial equation of degree n:

$$P(x) = 0.$$

Illustrate your algorithm by solving the equation:

$$x^4 + 3x^3 - 3x^2 - 11x - 6 = 0.$$

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3. Let $f(x)$ be an $(n+1)$ times continuously differentiable function of x and $f_o, f_1, \cdots, f_n$ are the values of $f(x)$ at $x = x_o, x_1, \cdots, x_n$ respectively.

(a) Derive the Lagrange's Interpolation polynomial $P_n(x)$ to estimate the value of $f(x)$ for any $x \in [x_o, x_n]$.

(b) The values of the function $f(x)$ are tabulated below:

x	0	0.5	1.5	2.0
f	0.500000	0.824361	2.240845	3.694528

Obtain a linear Interpolation polynomial and compute $f(0.75)$ and .

(c) Obtain a second order interpolation polynomial and compute $f(0.75$

(d) With the usual notations, show that the error in the interpolation is given by

$$E(x) = (x - x_o)(x - x_1) \cdots (x - x_n) \frac{f^{(n+1)}(\xi)}{(n+1)!}.$$

If $\left| f^{(n+1)}(x) \right| \leq M$ in $[x_o, x_n]$, obtain bounds on the errors in (b) and (c).

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4. Suppose you are required to compute

$$I = \int_a^b f(x)dx.$$

(a) Describe the **Trapezoidal method** to compute the value of I and derive a formula for the error. State the conditions that f should satisfy in order to apply Trapezoidal rule.

(b) With the usual notations, the **Simpson's rule** is given by

$$\int_{x_{i-1}}^{x_{i+1}} f(x)dx = \frac{h}{3}\left(f_{i-1} + 4f_i + f_{i+1}\right) - \frac{1}{90}h^5 f^{(iv)}(\eta_i), \quad \eta_i \in [x_{i-1},$$

Obtain the composite Simpson's rule and show that the composite error is

$$-\frac{1}{180}h^4(b-a)f^{(iv)}(\xi), \quad \xi \in [a, b].$$

(c) Describe **Romberg's** Integration method.

You should illustrate the use of Romberg table in calculating the successive values.