EASTERN UNIVERSITY, SRI LANKA

SECAND EXAAMINATION IN SCIENCE, 2001/2002

FIRST SEMESTER (April' 2002)

(Repeat)

ST 202 Distribution Theory

Answer All Questions

Time: 3 Hours

Q1.

(a) The probability density function of the Gamma distribution is

$$f(x)=\begin{cases}\dfrac{\lambda^{\alpha}x^{\alpha-1}e^{-\lambda x}}{\Gamma(\alpha)} & x>0\\ 0 & otherwise,\end{cases}$$

where $\alpha,\lambda>0$.

Show that the moment generating function of this distribution is $\left(\dfrac{\lambda}{\lambda-t}\right)^{\alpha}$ for $t<\lambda$ and hence find the mean and variance of the distribution.

(b) A sample of n values is drawn from a population whose probability density function is

$$f(x)=\begin{cases}e^{-x} & x>0\\ 0 & otherwise,\end{cases}$$

If $\bar{X}$ is the mean of the sample, show that $n\bar{X}$ has a Gamma distribution. What are the parameters of this distribution? Find the mean and variance of $\bar{X}$.

Q2.

A continuous random variable, X, has probability density function given by

$$f(x) = ax - bx^2 \quad for \quad 0 \le x \le 2$$
$$= 0 \quad \text{elsewhere.}$$

Observations on x indicate that the mean is 1.

(a) Show that a=1.5 and find the value of b.

(b) Find the variance of X.

(c) If F(x) is the probability that $X \le x$, find F(x) and verify that F(2)=1.

(d) If two independent observations are made on X, what is the probability that at least one of them is less than $\frac{1}{2}$?

Q3.

A machine is producing components whose lengths are normally distributed about a mean of 6.50 cm. An upper tolerance limit of 6.54 cm has been adopted and, when the machine is correctly set, 1 in 20 components is rejected as exceeding this limit. On a certain day, it is found that 1 in 15 components is rejected for exceeding this limit.

(a) Assuming that the mean has not changed but that the production has become more variables, estimate the new standard deviation.

(b) Assuming that the standard deviation has not changed but that the mean has moved, estimate the new mean.

(c) If 1000 components are produced in a shift, how many of them may be expected to have lengths in the range 6.48 to 6.53 cm if the machine is set as in (a)?

Q4.

(a) The number, X, of breakdowns per day of the lifts in a large block of flats has a Poisson distribution with mean 0.2. Find, to 3 decimal places, the probability that on a particular day

i) there will be at least one breakdown;
ii) there will be at most two breakdowns.

(b) Find, to 3 decimal places, the probability that, during a 20-day period, there will be no lift breakdowns.

(c) The maintenance contract for the lifts is given to a new company. With this company it is found that there are 2 breakdowns over a period of 30 days. Perform a significance test at the 5% level to decide whether or not the number of breakdown has decreased.

Q5.

(a) Define the **Joint probability density function** of random variables. Let X_1 and X_2 be a two independent standard normal random variables. Let $Y_1 = X_1 + X_2$, $Y_2 = \dfrac{X_1}{X_2}$. Find the joint probability density function of Y_1 and Y_2 and find the marginal distribution of Y_1 and Y_2.

(b) The random variable X has the probability density function

$$f(x) = \begin{cases} 2e^{-2x} & x > 0 \\ 0 & otherwise, \end{cases}$$

Find the moment generating function of X and hence find the mean and variance of x.

Show also that the median of the distribution is $\dfrac{1}{2}\log_e 2$ and the inter-quartile range is $\dfrac{1}{2}\log_e 3$.

Q6.

(a) A random sample of size 100 is taken from a normal population with variance $\sigma_1^2 = 40$. The sample mean $\bar{x}_1$ is 38.3. Another random sample, of size 80, is taken from a normal population with variance $\sigma_2^2 = 30$. The sample mean $\bar{x}_2$ is 40.1. Test, at the 5% level, whether there is a significant difference in the population means μ_1 and μ_2.

(b) It is claimed that the masses of components produced at a particular workshop are normally distributed with a mean mass of 6g and a standard deviation of 0.8g. If this claim is accepted, at the 5% level, on the basis of the mean mass obtained from a random sample of 50 components, between what values must the mean mass of the 50 components in the sample lie?