EASTERN UNIVERSITY, SRI LANKA

SECOND EXAMINATION IN SCIENCE 2003/2004

(June / July '2005)

SECOND SEMESTER

MT 205 - DIFFERENTIAL GEOMETRY

Answer all Questions

Time: one hour

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1. State **Serret-Frenet** formula.

(a) Show that the tangent vectors along the curve $\underline{r} = at\ \underline{e}_1 + bt^2\ \underline{e}_2 + t^3\ \underline{e}_3$ where $2b^2 = 3a$ make a constant angle with the vector $\underline{r}_1 = \underline{e}_1 + \underline{e}_3$.

(b) Define "osculating plane" and "rectifying plane" of a space curve. Find the equation for the osculating plane and rectifying plane to the curve $x = 3t - t^3$, $y = 3t^2$ and $z = 3t + t^3$ at the point $t = 1$.

(c) Show that for a curve lying on a sphere of radius a and such that the torsion τ is never 0, the following equation is satisfied

$$\left(\frac{1}{\kappa}\right)^2 + \left(\frac{\kappa'}{\kappa^2\tau}\right)^2 = a^2.$$

2. (a) Define the term "osculating circle of a space curve" and find it's radius and center.

If the curvature κ for a given curve C is constant. Then show that the curvature κ_1 for the locus of the centre of curvature of the osculating circle is also constant and is torsion varies inversely as that of C.

(b) Define the "Involute" and "Evolute" of a given curve C.

With usual notations find the vector equation of the involute of a given curve, $\underline{r} = \underline{r}(s)$. Then show that the torsion of the involute is given by

$$\frac{\kappa\tau' - \tau\kappa'}{\kappa(c-s)(\tau^2+\kappa^2)}.$$