EASTERN UNIVERSITY, SRI LANKA

SECOND EXAMINATION IN SCIENCE (2003/2004)

SECOND SEMESTER (JUNE/July.'2005)

MT 218 - FIELD THEORY

Repeat

Answer all questions Time:Two hours

1. State Gauss's theorem in the electro-static field.

(a) A charge q is uniformly distributed on a circle with equations $x^2 + y^2 = a^2, z = 0$. Show, with the usual notations that the potential at the point $P(0,0,z)$ is given by $\dfrac{q}{4\pi\epsilon_0\sqrt{a^2+z^2}}$.

Prove that the electric field at P is $\dfrac{qz}{4\pi\epsilon_0\left(a^2+z^2\right)^{\frac{3}{2}}}\underline{k}$.

(b) A spherical volume with radius a and charge density distribution ρ is given by

$$\rho = \begin{cases} \rho_0\left(1-\dfrac{r^2}{a^2}\right) & \text{if } \; r \leq a \\ 0 & \text{if } \; r > a. \end{cases}$$

i. Calculate the total charge.

ii. Find the electric field intensity outside of the charge distribution.

iii. Find the electric field intensity inside of the charge distribution.

2. (a) Define the term "electric dipole".

Prove that the electric potential V at a point P at a distance r form the dipole of moment $\underline{P}$ is given by

$$V = -\frac{1}{4\pi\varepsilon_0}\left\{\underline{P} \cdot grad\left(\frac{1}{r}\right)\right\}.$$

Hence prove that the force on a dipole in an electric field E is given by,

$$\underline{F} = (\underline{P} \cdot \nabla)\underline{E}$$

(b) What is dielectric polarization ?

Show, with the usual notation that the potential due to a finite volume of dielectric is given by

$$V = \frac{1}{4\pi\epsilon_0}\int_s \frac{\underline{P}.\underline{ds}}{r} + \frac{1}{4\pi\epsilon_0}\int_\tau \frac{-\text{div}\underline{\text{P}}}{r}\, d\tau$$

Interpret this result.

3. (a) Define the magnetic flux density $\underline{B}$ and show that div $\underline{B} = 0$ in space.

By assuming the Amphere's law in integral form deduce the equation Curl $\underline{B} = \mu_0 \underline{j}$, where $\underline{j}$ is the current density.

(b) Define the magnetic field strength $\underline{H}$ in a magnetizable media and show that Curl $\underline{H} = \underline{j}$.

In the absent of current, if the magnetization is linearly proportional to $\underline{H}$, show that there exists a function ϕ such that $\nabla^2\phi = 0$.

(c) A current I flows in a circular loop of wire of radius 'a'. Prove that the magnetic field at a point on the axis of the loop, at a distance z from its plane is directed along the axis and is of magnitude $\frac{Ia^2}{2(a^2+z^2)^{\frac{3}{2}}}$.

4. (a) Derive an expression for the velocity v that a particle strikes the earth when it drops at a height h from the ground of the earth.

(b) Show that the Poisson's equation $\nabla^2 U = \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dU}{dr}\right) = 4\pi\rho G$, for the gravitational potential U in a spherically symmetric distribution of matter having density ρ at a distance r from the center may be written as $\frac{1}{r}\frac{d^2}{dr^2}(rU) = 4\pi\rho G$.

A given spherical distribution of total mass M is given by,

$$\rho = \begin{cases} \rho_0 \dfrac{\sin\left(\frac{\pi r}{a}\right)}{\left(\frac{\pi r}{a}\right)} & \text{if } 0 \le r \le a \\ 0 & \text{if } r > a. \end{cases}$$

Show that $M = \frac{4\rho_0 a^3}{\pi}$.

Prove that $U = -\frac{GM}{a}\left(1 + \frac{\rho}{\rho_0}\right)$ for $r \le a$.

Calculate the self energy of the distribution in terms of G, H and a as compared with a state of infinite diffusion.