EASTERN UNIVERSITY, SRI LANKA
THIRD EXAMINATION IN SCIENCE - 2005/2006
SECOND SEMESTER(March/April, 2008)
MT 310 - FLUID MECHANICS
(PROPER AND REPEAT)

Answer all Questions **Time: Two hours**

Q1. (a) With the usual notation, write down the equation of continuity and the condition for a surface $F(r,t)=0$, where $r = x\underline{\mathbf{i}} + y\underline{\mathbf{j}} + z\underline{\mathbf{k}}$, to be a boundary surface. Hence, show that the surface

$$\left(\frac{x^2}{a^2}\right) f_1(t) + \left(\frac{y^2}{b^2}\right) f_2(t) + \left(\frac{z^2}{c^2}\right) f_3(t) = 1,$$

where f_1, f_2 and f_3 are functions of time t, and a, b and c are contants, is a possible boundary surface of an incompressible fluid if $f_1 f_2 f_3$ is a constant.

[60 marks]

(b) Obtain Cauchy-Reimann equations for an irrotational motion of an incompressible fluid in two-dimensions. If the stream function of this fluid is

$$(x-a)^2 - (y-b)^2 + c,$$

where a, b and c are constants, find the velocity potential. [40 marks]

Q2. (a) State and prove the Euler's equation of motion of an inviscid fluid. [30 marks]

(b) A rectangular coffee cup is placed on a turntable and rotated about its central axis with angular velocity Ω until a rigid body mode occurs. If the pressure at the point of intersection of the central axis and the base of the coffee cup is p_0, show that the free surface of equal pressure of the coffee is a parabolid revolution and the centre of the coffee drops by an amount

$$\frac{\Omega^2 R^2}{4g},$$

where $2R$ is the length of the base of the coffee cup and g is the gravitational acceleration. [55 marks]

Find the angular velocity which will cause the coffee to just reach the tip of the cup if $R = 3$cm, the height between the high and low points of free surface is 6cm and $g = 9.81\text{m/s}^2$. [15 marks]

Q3. (a) State the Milne-Thomson circle theorem. [10 marks]

(b) An incompressible fluid moves steadily and irrotationally under no external forces parallel to the $z-$plane past a fixed cylinder whose section in that plan is bounded by a closed curve C. If the complex potential for the flow is w, prove that the action of the fluid pressure on the cylinder is equivalent to a force per unit having components X and Y along the x and y-axes, respectively, and a moment M about the origin O, where

$$Y + iX = -\frac{1}{2}\rho \oint_C \left(\frac{dw}{dz}\right)^2 dz$$

and

$$M = Re\left\{-\frac{1}{2}\rho \oint_C z\left(\frac{dw}{dz}\right)^2 dz\right\}.$$

[40 marks]

A source, where rate of emission is k units of the volume per unit time, is at A such that $OA = f$, where f is real, outside a circular cylinder of radius a whose centre is at the origin O.

(i) Show that the image system for the source at A consists of an equal source at the inverse point of A and an equal sink at the centre O of the cylinder.

(ii) By finding the force acting on the cylinder, show that the cylinder will be attracted towards the source. [50 marks]

Q4. (a) If a solid boundary of a large spherical surface contains fluid in motion and encloses closed surfaces $S_m, m = 1, \ldots, k$, write down the equation for the kinetic energy of the moving fluid when it is at rest at infinity. [15 marks]

(b) If the velocity potential ϕ of such a fluid described in part (a) satisfies the Laplace's equation $\nabla^2\phi = 0$ and $\dfrac{\partial\phi}{\partial n}$ is a given function on $S_m, m = 1, \ldots, k$, show that ϕ is determined uniquely throughout a finite region. [35 marks]

(c) Suppose a puff of hot gas rises through air and it takes a roughly spherical shape. The air is sucked into the gas near the rear of the sphere which has fixed centre and radius a at time t. The mathematical model of the system is given by

$$\nabla^2\phi = 0, \quad r > a,$$

$$\phi \simeq Ur\cos\theta \quad \text{as} \quad r \longrightarrow \infty,$$

where r is the distance from the centre O of the puff to a point P in air and θ is the angle between OP and the direction of velocity of the air, with boundary condition

$$\left(\frac{\partial\phi}{\partial r}\right)_{r=a} = \frac{1}{5}\, v(1 - 3\cos\theta - 3\cos^2\theta),$$

where ϕ is the velocity potential, U is the constant velocity of the air and v is a constant. Show that

$$\phi = Ur\cos\theta + \frac{Ua^3}{2r^2}\cos\theta.$$

What is the significance of the above result?

In spherical polar coordinates (r, θ, ψ),

$$\nabla^2 \equiv \frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\left(\frac{\partial^2}{\partial\psi^2}\right)$$

and you may use the solution of $\nabla^2\phi = 0$ without proof. [50 marks]