EASTERN UNIVERSITY, SRI LANKA

FIRST EXAMINATION IN SCIENCE (2002/2003)

(April/May'2004)

Re-Repeat

MT 105 & 106 - PROBABILITY & STATISTICS

AND

DIFFERENTIAL EQUATIONS

Answer four questions only selecting two questions from each section

Time: Two hours

Section – A

1. Given the following frequency distribution.

Class interval	Frequency
15 – 25	4
25 – 35	11
35 – 45	19
45 – 55	14
55 – 65	6
65 – 75	2

(a) Draw the histogram and frequency polygon.

(b) Find the mean, median and mode for the above data.

(c) Find the mean deviation and variance.

2. (a) i. Define the term random variable.

ii. Consider an urn containing 5 red and 3 white balls. A random sample of size 3 is selected (without replacement). Let X be the number of red balls drawn. Find the probability mass function $f_X(x)$ and $P(0 \leq X \leq 3)$.

(b) i. A random variable X follows the Binomial distribution with parameters n and p. Show that

$$E(X) = np \quad \text{and} \quad \text{Var}(X) = npq, \quad \text{where} \quad q = 1 - p.$$

ii. The output of the production process is 10 percent defective. What is the probability of selecting at least 2 defective in a sample of size 5?

3. (a) State the Bayes's theorem.

(b) We assume that there are two urns available. The probability of choosing urn1 is $\frac{1}{10}$, for urn2, it is $\frac{9}{10}$. We suppose further that the urns contain black and white balls: in urn1, 70% of the balls are black, in urn2, 40% are black. What is the probability that a black ball is drawn blind folded is from urn1?

Section – B

4. (a) State the necessary and sufficient condition for the differential equation

$$M(x,y)dx + N(x,y)dy = 0$$

to be exact.

Solve the following differential equation:

$$(y^2 - x^2 \sin xy)\frac{dy}{dx} = xy \sin xy - \cos xy - e^{2x}.$$

(b) Show that if,

$$\frac{1}{M(x,y)}\left(\frac{\partial N(x,y)}{\partial x} - \frac{\partial M(x,y)}{\partial y}\right)$$

is a function of y only and equal to $f(y)$ then, $e^{\int f(y)dy}$ is the integrating factor of $M(x,y)dx + N(x,y)dy = 0$.

Hence solve the following differential equation

$$3x^2y^2dx + 4(x^3y - 3)dy = 0.$$

5. (a) $F(D) = p_0D^n + p_1D^{n-1} + \cdots + p_{n-1}D + p_n$ and V be a function of x only, where $D = \dfrac{d}{dx}$ and $p_0, p_1, \cdots, p_n$ are constants. Prove that

i. $F(D)xV = xF(D)V + F'(D)V$,

ii. $\dfrac{1}{F(D)}xV = x\dfrac{1}{F(D)}V - \dfrac{F'(D)}{[F(D)]^2}V$, provided $F(D) \neq 0$.

(b) Solve the following differential equations:

i. $(D^2 + 9)\,y = x\cos x$,

ii. $(D^2 - 6D + 13)\,y = 8e^{3x}\sin 2x$.

6. (a) Find the general solution of the following differential equations:

i. $(x^3D^3 + 2x^2D^2)\,y = x + \sin(\ln x)$,

ii. $(1+x)^2\dfrac{d^2y}{dx^2} + (1+x)\dfrac{dy}{dx} + y = \cos[\log(1+x)]$.

(b) Solve the following simultaneous differential equation:

$$(D-3)\,x + 2\,(D+2)\,y = 2\sin t,$$

$$2\,(D+1)\,x + (D-1)\,y = \cos t,$$

where $D = \dfrac{d}{dt}$.