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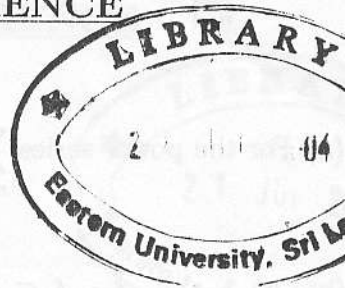
EASTERN UNIVERSITY, SRI LANKA  
FIRST YEAR EXAMINATION IN SCIENCE

2002/2003 & 2002/2003 (A)

SECOND SEMESTER

(April/May '2004)

MT 105 - THEORY OF SERIES



Answer All Questions

Time: 1 Hour

Q1. (a) Define what is meant by the infinite series  $\sum_{n=1}^{\infty} a_n$  is convergent.

[5 Marks]

Show that the series

$$\sum_{n=1}^{\infty} \frac{1}{n(n+4)} = \frac{1}{1.5} + \frac{1}{2.6} + \frac{1}{3.7} + \dots$$

is convergent and find its sum.

[30 Marks]

(b) State the theorem of **Integral Test**.

[10 Marks]

By using the above theorem or otherwise, for the following cases of  $p \in \mathbb{R}$ ,

(i)  $p > 1$ ,

(ii)  $p = 1$ ,

(iii)  $0 < p < 1$ ,

determine whether the series  $\sum_{n=1}^{\infty} \frac{1}{n^p}$  converges or diverges.

[15 Marks]

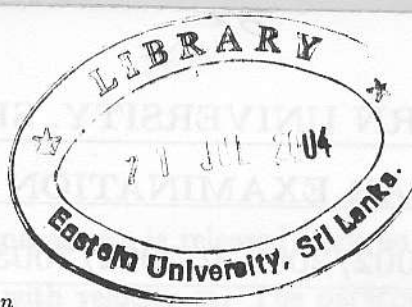
(c) State the theorem of **Alternating Series Test**.

[10 Marks]

Use the above theorem to decide whether the following series converge or diverge:

$$(i) \sum_{n=1}^{\infty} \sin \left( \frac{(n^2 + 1)\pi}{n} \right)$$

$$(ii) \sum_{n=1}^{\infty} (-1)^{n+1} \left( \frac{n+1}{n(n+2)} \right)$$



[30 Marks]

2. (a) For the power series  $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n}$ , find the interval and radius of convergence.

[25 Marks]

(b) (i) Let  $f_n, f : A \subseteq \mathbb{R} \rightarrow \mathbb{R}$ . Define what is meant by  $f_n \rightarrow f$  as  $n \rightarrow \infty$  uniformly on  $A$ .

[5 Marks]

(ii) Let  $f_n, f : A \subseteq \mathbb{R} \rightarrow \mathbb{R}$ . If  $f_n \rightarrow f$  uniformly on  $A$  as  $n \rightarrow \infty$  and each  $f_n, n \in \mathbb{N}$  is continuous on  $A$ , then prove that  $f$  is continuous on  $A$ .

[20 Marks]

(iii) Let  $f_n, f : [a, b] \subseteq \mathbb{R} \rightarrow \mathbb{R}$  and let  $f_n \rightarrow f$  uniformly on  $[a, b]$  as  $n \rightarrow \infty$  and each  $f_n, n \in \mathbb{N}$  is continuous on  $[a, b]$ . Show that

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \int_a^b f_n(x) dx.$$

[20 Marks]

(c) (i) Show that

$$\ln x = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{(x-1)^n}{n} \quad \text{for } |x-1| < 1.$$

[15 Marks]

(ii) Use the result in part(i) and the Abel's theorem to show that

$$\ln 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$$

[15 Marks]