EASTERN UNIVERSITY, SRI LANKA

FIRST YEAR EXAMINATION IN SCIENCE

2003/2004

SECOND SEMESTER

(June/July - 2005)

Proper & Repeat

MT 105 - THEORY OF SERIES

Answer **All** Questions — Time: 1 Hour

Q1. (a) Define what is meant by the infinite series $\sum_{n=1}^{\infty} a_n$ is convergent.

[5 Marks]

Show that the series

$$\sum_{n=1}^{\infty} \frac{1}{(4n-1)(4n+3)} = \frac{1}{3.7} + \frac{1}{7.11} + \frac{1}{11.15} + \dots$$

is convergent and find its sum.

[30 Marks]

(b) State the theorem of **Integral Test**.

[10 Marks]

By using the above theorem or otherwise, for the following cases of $p \in \mathbb{R}$,

(i) $p > 1$,

(ii) $p = 1$,

(iii) $0 < p < 1$,

determine whether the series $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^p}$ converges or diverges.

[15 Marks]

(c) State the theorem of **Alternating Series Test.**

[10 Marks]

Use the above theorem to decide whether the following series converge or diverge:

(i) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2^n}$;

(ii) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\, n}{(3n-1)}$.

[30 Marks]

Q2. (a) For the power series $\displaystyle\sum_{n=1}^{\infty} \frac{n(x-1)^n}{2^n(3n-1)}$, find the interval and radius of convergence.

[25 Marks]

(b) (i) Let $f_n, f : A \subseteq \mathbb{R} \to \mathbb{R}$. Define what is meant by $f_n \to f$ as $n \to \infty$ uniformly on A. [5 Marks]

(ii) Let $f_n, f : A \subseteq \mathbb{R} \to \mathbb{R}$. If $f_n \to f$ uniformly on A as $n \to \infty$ and each f_n, $n \in \mathbb{N}$ is continuous on A, then prove that f is continuous on A.

[20 Marks]

(iii) Let $f_n, f : [a,b] \subseteq \mathbb{R} \to \mathbb{R}$ and let $f_n \to f$ uniformly on $[a,b]$ as $n \to \infty$ and each f_n, $n \in \mathbb{N}$ be continuous on $[a,b]$. Show that

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \int_a^b f_n(x)\,dx.$$

[20 Marks]

(c) (i) Show that

$$\arctan x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} \quad \text{for } |x| < 1.$$

[15 Marks]

(ii) Use the result in part(i) and the **Abel**'s theorem to show that

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$$

[15 Marks]