EASTERN UNIVERSITY, SRI LANKA

THIRD EXAMINATION IN SCIENCE

(2002/03 & 2002/03(A))

SECOND SEMESTER (Apr./May '2004)

MT 310 - FLUID MECHANICS

Answer all questions | Time: Two hours

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1. With the usual notation, derive the continuity equation for a fluid flow in the form

$$\frac{d\rho}{dt} + \rho \, div \, \mathbf{q} = 0$$

where $\frac{d}{dt}$ denotes the differentiation following a fluid particle. Using the relations $x = r\cos\theta$, $y = r\sin\theta$, express the vector

$$\mathbf{q} = \frac{c(-y\mathbf{i} + x\mathbf{j})}{x^2 + y^2},$$

in cylindrical polar coordinates (r, θ, z). Hence show that

(a) the motion of an incompressible fluid is possible, with velocity $\mathbf{q}$, and that the streamlines form a family of circles with centers on the $oz-$ axis.

(b) this motion is irrotational with velocity potential $\phi = -c\theta$

(c) streamlines intersect equipotential surfaces orthogonally

(d) the circulation of velocity around any curve in the $oxy-$ plane is $2\pi c$.

2. With the usual notation, derive the equation of motion of an inviscid fluid in the form

$$\frac{d\mathbf{q}}{dt} = \mathbf{F} - \frac{1}{\rho}\nabla p$$

A sphere of radius $R(t)$ whose center is at rest, vibrates radially in an infinite incompressible liquid of constant density ρ. The liquid, which is under no external body force, extends to infinity, where it is at rest. Show that the motion of the liquid is irrotional with velocity potential

$$\phi = \frac{R^2\dot{R}}{r} \quad \text{where } \dot{R} = \frac{dR}{dt}.$$

If the pressure at infinity is p_∞, show that the pressure at the surface of the sphere $(r = R)$, at time t, is

$$p = p_\infty + \rho\left[R\ddot{R} + \frac{3}{2}\dot{R}^2\right].$$

If $R = a + b\sin nt$ where a, b, n are constants such that $a > b$, show that in order that there is no cavitation $p_\infty \geq \rho n^2 b(a+b)$.

3. State and prove the Circle theorem for an irrational two-dimensional flow of an incompressible inviscid fluid moving parallel to $xy-$ plane.

A two dimensional source of strength m is placed at a point $C(z = c)$ outside a fixed circular boundary of centre O and radius a. By finding the image system or otherwise, find the complex velocity at any point $P(z)$ where $|\, z \,| \geq a$.

Show that the magnitude of the velocity is $\frac{\text{m.AP.BP}}{\text{OP.CP.DP}}$, where A,B are the points where OC cuts the circle and D is the inverse point of C.

4. For an incompressible fluid in irrotational motion under conservative forces obtain the pressure equation

$$\frac{p}{\rho}+\frac{1}{2}q^2+\Omega-\frac{\partial\phi}{\partial t}=f(t).$$

with the usual notation.

A solid sphere of radius a, uniform density σ moves in a straight line with velocity $U(t)$ through an infinite volume of liquid with uniform density ρ which is at rest at infinity. Show that

$$\frac{dU}{dt}=\frac{F}{\left(M+\frac{M'}{2}\right)},$$

where F is the external force acting on the sphere and $M=\frac{4}{3}\pi a^3\sigma$ and $M'=\frac{4}{3}\pi a^3\rho$.

Hence deduce that acceleration of a sphere falling vertically in an infinite fluid which is at rest at infinity is

$$\frac{g(\sigma-\rho)}{\left(\sigma+\frac{\rho}{2}\right)}.$$