EASTERN UNIVERSITY, SRI LANKA,

SECOND EXAMINATION IN SCIENCE - 2004/2005

(Oct./Nov.' 2006)

SECOND SEMESTER

ST 203 - EXPERIMENTAL DESIGN

Answer all questions

Time : Three hours

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1. The Mathematical model for the data consisting of k independent random samples of size n is

$y_{ij} = \mu + \tau_i + \epsilon_{ij}$; $\quad i = 1, 2, \cdots, k$ and $j = 1, 2, \cdots, n$

where μ is the overall mean, τ_i is the deviation of the i^{th} population mean from the overall mean, ϵ_{ij} is the error terms which are normally and independently distributed with mean zero and variance σ^2.

(a) Show that

$$\sum_i \sum_j (y_{ij} - y_{..})^2 = \sum_i \sum_j (y_{ij} - y_{i.})^2 + n \sum_i (y_{i.} - y_{..})^2$$

where $y_{..} = \sum_i \sum_j \frac{y_{ij}}{nk}$, $\quad y_{i.} = \sum_j \frac{y_{ij}}{n}$.

(b) Find the distribution of

$$\frac{\sum_i \sum_j (y_{ij} - y_{..})^2}{\sigma^2}$$

and state the assumption that you used.

(c) Show that

$$\frac{\sum_i \sum_j (y_{ij} - y_{i.})^2}{k(n-1)}$$

is an unbiased estimator for σ^2.

(d) Find the distribution of $\frac{T}{E}$

where $T = \frac{n}{k-1} \sum_i (y_{i.} - y_{..})^2$

and

$$E = \frac{1}{k(n-1)} \sum_i \sum_j (y_{ij} - y_{..})^2$$

Q2. Four different paints were compared for their ability to protect ships in a sea-going environment. Sixteen ships were used, each painted with one of the four paints. After the ships were in use for six months a score was assigned to the state of the paint in each ship. A higher score indicated a better state. The scores are given in the following table.

Paint 1	80	73	72	90
Paint 2	81	82	88	84
Paint 3	93	80	80	97
Paint 4	89	86	96	99

(i) Carry out a one-way analysis of variance of these data stating the assumption you have made null and alternative hypotheses and explain your conclusion.
(ii) Suppose each column of the above table represents a different geographical area in which the ships were used. Using this additional information re-analysis the data and comment upon whether your conclusions are affected by this additional information.

Q3. The lifetime of an electronic component in a standard test program depends on two factors involved in its manufacture: S, the source of supply of one of the raw materials and T, the temperature at which one part of the manufacturing process is operated. There are two sources of material (S_1 and S_2) and two temperatures (T_1 = 250°C and T_2 = 300°C) and five components are available for each of the four combinations of levels of S and T. The components are tested in a randomized-block scheme, blocks being different days of starting the test. The lifetimes (hours) were as follows:

		Day					
Level of S	Level of T	I	II	III	IV	V	Total
S_1	250°C	5.3	6.0	4.8	5.2	5.7	27
S_1	300°C	10.9	12.5	11.2	10.1	11.5	56.2
S_2	250°C	18.4	17.7	19.0	16.3	18.9	90.3
S_2	300°C	28.7	26.6	27.5	28.0	27.5	138.3
	Total	63.3	62.8	62.5	59.6	63.6	311.8

The sum of the squares of all the observations, $\sum y_{ijk}^2$ is 6244.92.

Carry out an appropriate analysis and report on its results.
Illustrate the results on a graph which shows the mean of each (S,T) combination and explain how the graph helps in understanding the results.

Q4. (a) What is the difference between completely randomized design and randomized complete block design?
(b) In an experiment to study the yields of five varieties of paddy, four blocks of land were used. Each block was divided into five sub–blocks. Each of the five varieties was used in each of the five sub-blocks. The following table gives the yields of paddy (in bushels):

		Varieties of paddy				
		1	2	3	4	5
Block	1	45	40	30	35	53
	2	40	42	37	45	48
	3	46	38	26	37	35
	4	38	42	25	30	32

(i) Suggest a suitable model to analyze this data and the assumptions.
(ii) Construct the ANOVA table.
(iii) Test whether the varieties are equally good with respect to the yield and the best variety if they are not equally good.
(iv) Do the data indicate that the effects in yield differ in blocks?

Q5. In an experiment to study the effect of four treatments (A, B, C, D), each rabbit was given a treatment on each of four days. The results recorded in the form of Latin Square Design are given below.

	Rabbit Number			
Day	1	2	3	4
1	B 24	C 46	D 34	A 48
2	D 33	A 58	B 57	C 60
3	A 57	D 26	C 60	B 45
4	C 46	B 34	A 61	D 47

(i) Analyze the data
(ii) Set up another similar field plan for this experiment

Q6. An engineer is studying the effect of deposition time and deposition temperature on the thickness of a layer of deposit. He used two setting of deposition time (Low and High relative to currently used time) and two temperatures (1210° C and 1240° C). Measurement of thickness of the layer was made on each of five deposits produced under each set of conditions. The measurements are given in table below.

		Deposition Time	
		High	Low
Deposition Temperature	1210° C	14.90,14.69,14.52, 15.14,14.63	13.78,14.18,13.58, 13.58, 13.81
	1240° C	14.49,14.33,13.94, 14.31,14.18	14.27, 14.37,14.16, 14.03,14.20

The sum of the 20 measurements is 285.09 and corresponding sum of squares is 4067.00. Perform an analysis of variance.
Draw a diagram of means to explain any interaction.