EASTERN UNIVERSITY, SRI LANKA,
SECOND EXAMINATION IN SCIENCE - 2004/2005
(Oct./Nov.' 2006)
SECOND SEMESTER
ST 204 - STATISTICAL INFERENCE II

Answer all questions

Time : Two hours

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1. A company sells detergent packed in two machines. From past experience, the company knows that the amount of detergent boxes packed in the two machines are normally distributed. The company takes a random sample of 25 boxes from the output of each machine and finds that the mean weight and standard deviation of the detergent in the boxes from machine 1 is 1064gms and 100gms respectively. For the sample in machine 2, the mean is 1024gms and standard deviation is 70gms.

   (a) Can the company claim with 5% level of significance that the boxes of detergent from machine 1 contain more than 1000gms.

   (b) Test at the 5% level of significane that the amount of detergent the boxes of both machines is same.

2. (a) Define the following terms:

   i. Type I error and Type II error,

   ii. Critical region,

   iii. Power function.

   (b) Let $X_1, X_2, \cdots, X_n$ be a random sample from a normal distribution with mean μ and variance 4. Let the critical region for testing $H_0 : \mu = 1$ versus $H_1 : \mu = 2$ be $\{\underline{X} : \sum_{i=1}^{n} X_i > k\}$. If $\alpha = 0.05$ and $\beta = 0.01$, find the values of n and k, and deduce the critical region, where α and β are the probabilities of Type I and Type II errors respectively.

3. (a) Describe the Neyman-Pearson approach to testing one simple hypothesis against another simple hypothesis.

(b) Suppose that $X_1, X_2, \cdots, X_n$ are independent random variables such that X_i has a normal distribution with mean θ_i and variance 1. It is required to test the null hypothesis that each θ_i is zero against the alternative hypothesis that $\theta_i = \frac{1}{2}$ for $i = 1, 2, \cdots, r$ and $\theta_i = -\frac{1}{2}$ for $i = r+1, r+2, \cdots, n$.

(i) Show that the most powerful test has critical region depending on the value of $\sum_{i=1}^{r} X_i - \sum_{i=r+1}^{n} X_i$.

(ii) Find the most powerful test with size 0.05.

(iii) Evaluate the power of the test found in (ii) and how large n must be to ensure that the power is at least 0.9.

4. (a) Explain what is meant by a minimax decision rule.

(b) Each item produced by a machine is subjected to a quick test which has three results: r_1(too small), r_2(correct size) and r_3(too big). If the item really is the correct size, the probabilities of these results are $P(r_1) = 0.1$, $P(r_2) = 0.7$ and $P(r_3) = 0.2$, while if it is wrong size the probabilities are $P(r_1) = 0.4$, $P(r_2) = 0.3$ and $P(r_3) = 0.3$. After each item is tested it is either sold or scrapped. If an item of incorrect size is sold, there is a penalty cost of Rs10, while if an item is scrapped a cost of Rs3 is incurred.

i. List the possible decision rules for deciding whether each item should be scrapped

ii. Calculate the risk table and find the minimax decision rule.

iii. If the prior information, the probability of the item really correct size is 0.6 which is the best of these strategies.