EASTERN UNIVERSITY, SRI LANKA

THIRD EXAMINATION IN SCIENCE 2003/2004

(June/July' 2005)

SECOND SEMESTER

MT 304 - GENERAL TOPOLOGY

Answer all questions

Time: Two hours

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1. Define the following terms:

   - Topology on a set,
   - Subspace of a topology,
   - Base for a topology.

   (a) Let X be a non-empty set. Let τ be the collection of subsets of X consisting of the empty set Φ and all subsets whose complements are finite. Is (X, τ) a topological space? Justify your answer.

   (b) Let (Y, τ_Y) be a subspace of a topological space (X, τ). Prove that $A \subseteq Y$ is closed in (Y, τ_Y) if, and only if, $A = F \cap Y$ for some closed subset F of X in (X, τ).

   (c) Let $\mathbf{B}$ be a base for a topology τ on X and $S \subseteq X$. Prove that the collection $B_S = \{U \cap S \,:\, U \in \mathbf{B}\}$ is a base for the subspace topology τ_S on S.

2. Let f be a function from a topological space (X, τ_1) into a topological space (Y, τ_2). What is meant by that f is continuous at a point $x_0 \in X$?

Let f be a function from a topological space (X, τ_1) into a topological space (Y, τ_2). Prove the following.

(a) f is continuous if, and only if, $f^{-1}(F)$ is closed in X, for each closed set F in Y.

(b) f is continuous if, and only if, $f(\overline{A}) \subseteq \overline{f(A)}, \quad \forall\, A \subseteq X$.

(c) If f is continuous then for every compact subset A of X the image $f(A)$ is compact in Y.

3. Let (X, τ) be a topological space. Prove that the following statements are equivalent.

(i) X is connected.

(ii) X cannot be expressed as the union of two disjoint non-empty closed sets.

(iii) The only non-empty subset of X which is both open and closed is X itself.

(iv) The set of all Frontier points of A (Fr A) is non-empty, for any non-empty proper subset of X.

(v) There is no continuous function from X onto Y, when $Y = \{0, 1\}$ has the discrete topology.

4. Define the following terms:

- Frechet space (T_1),
- Housdorff space,
- Compact set.

(a) Prove that a closed subset of a compact topological space is compact.

(b) Let A be a compact subset of a Housdorff topological space X and let $p \in X \setminus A$. Prove that there exist open sets G and H such that $p \in G, \quad A \subseteq H$ and $G \cap H = \phi$.

(c) Prove that every compact subset of a Housdorff topological space is closed.

(d) Is it true that every Frechet space (T_1) is Housdorff space? Justify your answer.