EASTERN UNIVERSITY, SRI LANKA

THIRD EXAMINATION IN SCIENCE(2003/2004)

SECOND SEMESTER

(June/July'2005)

MT - 307 CLASSICAL MECHANICS III

Proper & Repeat

Answer all questions

Time:Three hours

1. Prove that the velocity $\frac{d\underline{r}}{dt}$, relative to the fixed frame, of a particle P in a rotating frame can be written in the form

$$\frac{d\underline{r}}{dt}=\frac{\partial \underline{r}}{\partial t}+\underline{\omega}\wedge \underline{r},$$

where $\underline{\omega}$ is the angular velocity of the rotating frame and $\frac{\partial \underline{r}}{\partial t}$ is the velocity of P relative to the rotating frame. Hence show that the acceleration of P is given by

$$\frac{d^2\underline{r}}{dt^2}=\frac{\partial^2 \underline{r}}{\partial t^2}+2\underline{\omega}\wedge\frac{\partial \underline{r}}{\partial t}+\frac{\partial \underline{\omega}}{\partial t}\wedge \underline{r}+\underline{\omega}\wedge(\underline{\omega}\wedge\underline{r}).$$

If a projectile is fired due east from a point on the earth's surface at the northern latitude λ , with a velocity of magnitude V_0 and at an angle of inclination α to the horizontal, show that the lateral deflection d, when the projectile strikes the earth, is given by

$$d=\frac{4V_0^3}{g^2}\omega\sin\lambda\sin^2\alpha\cos\alpha,$$

where $\underline{\omega}$ is the angular velocity of the earth, terms of $O(\omega^2)$ being negligible. If the range of the projectile is R for the case $\omega=0$, show that the change of range due to the rotation of the earth is

$$\Delta R=\sqrt{\frac{2R^3}{g}}\omega\cos\lambda\left\{\cot^{\frac{1}{2}}\alpha-\frac{1}{3}\tan^{\frac{3}{2}}\alpha\right\}.$$

2. (a) With the usual notation, obtain the equations

i. $\dfrac{d\underline{H}}{dt} = \sum_{i=1}^{N} \underline{r}_i \wedge \underline{F}_i$,

ii. $\dfrac{d\underline{H}_G}{dt} = \sum_{i=1}^{N} \underline{R}_i \wedge \underline{F}_i$ for a system of N particles moving in space.

(b) A sphere of mass m and radius b is at rest upon a fixed sphere of radius $a(> b)$. The upper sphere is moved slightly to roll under the influence of gravity.The coefficient of static friction is $\mu_s > 0$, the coefficient of sliding friction is $\mu = 0$. At time t, θ is the angle of inclination of center of the upper sphere relative to the vertical axis through the center of the fixed sphere.

i. Write the equation of the motion in terms of $\ddot{\theta}$ and θ for the motion of the sphere rolls without slipping.

ii. Find $\dot{\theta}$ in terms of θ.

iii. Solve this equation for $\theta(t)$, assuming $0 < \theta(0) \ll \theta(t)$. You may use the relation

$$\int \frac{dx}{\sin(\frac{x}{2})} = 2 \ln \tan\left(\frac{x}{4}\right).$$

3. With the usual notation obtain the Euler's equations for the motion of a rigid body, with one point fixed, in the form:

$$A\dot{\omega}_1 - (B-C)\omega_2\omega_3 = N_1,$$
$$B\dot{\omega}_2 - (C-A)\omega_1\omega_3 = N_2,$$
$$C\dot{\omega}_3 - (A-B)\omega_1\omega_2 = N_3.$$

A solid consists of two equal uniform right circular cones each having height b and with their vertices rigidly jointed at O such that their axes in the same straight line, the vertex angle of each cone being $\frac{\Pi}{2}$.
If O is fixed and the solid is set to rotate about a common generator of the cones with angular velocity Ω, under no forces except gravity and reaction at O, show that the solid will rotate about the same generator after a time $\frac{10\Pi\sqrt{2}}{3\Omega}$ if the principle moments of inertia are $A = B = \frac{3}{4}Mb^2$ and $C = \frac{3}{10}Mb^2$.

4. Obtain the Lagrange's equation from the D'Alembert principle for a holonomic system.

A sphere of mass M and radius R rolls without slipping down on the inclined plane of a wedge shaped block of mass m that is free to move on a frictionless horizontal surface.

(a) Find the Lagrange's equations for this system subject to the force of gravity at the surface of the earth.

(b) Find the motion of the system by integrating Lagrange's equations, given that all objects are initially at rest and the center of the sphere is at a distance H above the surface.

5. (a) Define the Hamiltonian of a holonomic system in terms of its Lagrangian. Consider the system of particles m_1, m_2 $(m_1 = m_2)$ connected by a weightless rope of length l with m_2 constrained to move on the frictionless surface of an upright cone of half angle α and m_1 hanging freely inside the cone while the rope passing through a small hole at the top of the cone.

    i. Write down the Lagrangian of the system.

    ii. Find the Hamiltonian function of the system.

   (b) Define the Poission bracket.

   With the usual notation prove that

    i. $[f, g+h] = [f, g] + [f, h]$,

    ii. $[f, q_j] = -\dfrac{\partial f}{\partial p_j}$,

    iii. $[f, p_j] = \dfrac{\partial f}{\partial q_j}$.

6. With the usual notations, derive Lagrange's equation for the impulsive motion from Lagrange's equations for a holonomic system in the following form

$$\Delta\left(\frac{\partial T}{\partial \dot{q}_j}\right) = \sum_i I_i \frac{\partial \underline{r}_r}{\partial q_j} \qquad j = 1, 2, \ldots n.$$

A square $ABCD$ formed by four equal rods, each of length $2l$ and mass m joined smoothly at their ends, rests on a smooth horizontal table. An impulse of magnitude I is applied to the vertex A in the direction AD.

   (a) Find the equation of motion of the frame.

   (b) Show that the kinetic energy of the square immediately after the application of impulse is $\dfrac{5I^2}{16m}$.