# EASTERN UNIVERSITY, SRI LANKA

## THIRD EXAMINATION IN SCIENCE - 2004/2005

## FIRST SEMESTER (Jan./ Feb., 2006)

## MT 304 - GENERAL TOPOLOGY

Answer all questions

Time allowed : Two hours

1. (a) Define the term **topological space.**

   Let $X$ be a non-empty set and let $\tau$ be the family consisting the empty set $\Phi$ and all those non-empty subsets of $X$, whose complements are countable. Show that $(X, \tau)$ is a topological space.

   (b) Let $A$ be a non-empty subset of a topological space $(X, \tau)$. Prove the following:

   i. $\overline{A}$ is the smallest closed superset of $A$,

   ii. $\overline{A} = A \cup A'$,

   iii. $x \in \overline{A}$ if and only if $\forall\, G \in \tau$ with $x \in G$ and $G \cap A \neq \Phi$,

   iv. $\mathrm{Fr}(A) = \overline{A} \cap \overline{A^c}$, where $\mathrm{Fr}(A)$ is the set of all frontier points of $A$.

2. Define the term **disconnected set** in a topological space.

   Prove the following:

   (a) A topological space $(X, \tau)$ is disconnected if and only if there exist a non-empty proper subset of $X$ which is both open and closed,

   (b) A topological space $(X, \tau)$ is disconnected if and only if there exist a non-empty proper subset $A$ of $X$ such that $\mathrm{Fr}(A) = \Phi$,

   (c) Continuous image of a connected set is connected.

3. Let $f$ be a function from a topological space $(X, \tau_X)$ to a topological space $(Y; \tau_Y)$. Prove that the following statements are equivalent:

(a) $f$ is continuous.

(b) $f^{-1}(G)$ is open in $X$ whenever $G$ is open in $Y$.

(c) $f^{-1}(H)$ is closed in $X$ whenever $H$ is closed in $Y$.

(d) $f(\overline{A}) \subseteq \overline{f(A)} \qquad \forall A \subseteq X$.

4. Define the **Frechet space** $(T_1)$ and the **Hausdorff space** $(T_2)$.

(a) Prove that every Hausdorff space is Frechet space.

Is the converse true? Justify your answer.

(b) If $A$ be a non-empty proper compact subset of a Hausdorff space then prove that $A$ is closed.

(c) If $A$ and $B$ are two non-empty disjoint proper compact subsets of a Hausdorff space then show that there exist two disjoint open subsets $G$ and $H$ such that $A \subseteq G$ and $B \subseteq H$.