EASTERN UNIVERSITY, SRI LANKA

THIRD EXAMINATION IN SCIENCE - 2004/2005

FIRST SEMESTER (Mar./ Apr., 2006)

REPEAT

MT 307 - CLASSICAL MECHANICS III

Answer all questions **Time : 03 hours**

1. Two frames of reference S and S' have a common origin O and S' rotates with constant angular velocity $\underline{\omega}$ with respect to S. At a time t a particle P has position vector $\underline{r}$ with respect to O. Prove that the acceleration of P relative to S is

$$\ddot{\underline{r}} + +2\underline{\omega} \wedge \dot{\underline{r}} + \underline{\omega} \wedge (\underline{\omega} \wedge \underline{r}),$$

where $\dot{\underline{r}}$, $\ddot{\underline{r}}$ denote respectively the velocity and the acceleration of P relative to S'.

An object of mass m initially at rest is dropped to the earth's surface from a height h above the earth's surface. Assuming that the angular speed of the earth about its axis is a constant w, prove that after time t the object is deflected east of the vertical by the amount

$$\frac{1}{3}\,\omega g t^3 \cos\lambda,$$

and show that it hits the earth at a point east of the vertical at a distance

$$\frac{2}{3}\,\omega h \cos\lambda\,\sqrt{\frac{2h}{g}},$$

where λ is the earth's latitude.

2. (a) With the usual notation, obtain the equations:

i. $\dfrac{d\underline{H}}{dt} = \sum_{i=1}^{N} \underline{r}_i \wedge \underline{F}_i$,

ii. $\dfrac{d\underline{H}_G}{dt} = \sum_{i=1}^{N} \underline{R}_i \wedge \underline{F}_i$, for a system of N particles moving in space.

(b) a sphere of mass m, radius a , and moment of inertia $\dfrac{2}{5}\, ma^2$ rolls without slipping from its initial position at rest from a top of a fixed cylinder of radius b, which lies on a horizontal plane.

i. Determine the angle of maximum at which the sphere leaves the cylinder.

ii. What are the components of the velocity of the sphere's center at the instant it leaves the cylinder.

3. With the usual notation, obtain the Euler's equation's for the motion of a rigid body, having a point fixed, in the form

$$A\dot{\omega}_1 - (B - C)\omega_2\omega_3 = N_1,$$
$$B\dot{\omega}_2 - (C - A)\omega_1\omega_3 = N_2,$$
$$C\dot{\omega}_3 - (A - B)\omega_1\omega_2 = N_3.$$

A rigid body having an axis of symmetry OC moves about a fixed point O under no forces except a constant retarding couple μC about the axis OC. If A, A, C are the principal moments of inertia and $(\omega_1, \omega_2, \omega_3)$ the components of the angular velocity about the principal axes OA, OB and OC, show that, after a time t,

$$\omega_1 = \Omega \cos\left[pt\left(\Omega - \left(\frac{\mu}{2}\right)t\right)\right]$$
$$\omega_2 = -\Omega \sin\left[pt\left(\Omega - \left(\frac{\mu}{2}\right)t\right)\right] \quad \text{and}$$
$$\omega_3 = \Omega - \mu t$$

where $p = \dfrac{(A - C)}{A}$ and $(\Omega, 0, \Omega)$ is the initial angular velocity of the rigid body.

4. (a) Define Hamiltonian H in terms of the Lagrangian L.

Show, with usual notation, that the Hamilton's equations are given by

$$\dot{q}_j = \frac{\partial H}{\partial p_j}, \qquad \dot{p}_j = -\frac{\partial H}{\partial q_j} \quad \text{and} \quad \frac{\partial \mathrm{H}}{\partial \mathrm{t}} = -\frac{\partial \mathrm{L}}{\partial \mathrm{t}}.$$

Show that if the Lagrangian is not an explicit function of time and the system is conservative, then the Hamiltonian H is a constant and equal to the total kinetic energy of the system.

(b) A particle of mass m can slide without friction on the inside of a small tube bent in the form of a circle of radius r. The tube can rotate freely about a vertical axis and has a moment of inertia I about this axis. Find the Hamiltonian's equations of motion for the system.

5. With usual notations deduce Lagrange's equations for impulsive motion from Lagrange's equations for a holonomic system in the form

$$\left(\frac{\partial T}{\partial \dot{q}_j}\right)_2 - \left(\frac{\partial T}{\partial \dot{q}_j}\right)_1 = S_j, \qquad j = 1, 2, \cdots, n$$

where subscripts 1 and 2 denote quantities before and after the application of the impulse respectively.

A uniform rod AB of length $2a$ and mass m has a particle of mass M attached to the end B. It is at rest on a smooth horizontal table when an impulse I is applied at A in a direction perpendicular to AB, and in the plane of the table. Find the initial velocities of A and B and prove that the resulting kinetic energy is

$$\frac{2I^2(m+3M)}{(m^2+4mM)}.$$

6. Define the Poisson bracket.

(a) With the usual notations prove that

i. $[fg, h] = [f, h]g + f[g, h]$

ii. $[f, q_k] = -\dfrac{\partial f}{\partial p_k}$ and

iii. $[f, p_k] = \dfrac{\partial f}{\partial q_k}$.

(b) Show that the Hamiltonian equations of a holonomic system may be written in the form

$$\dot{q}_k = [q_k, H], \quad \dot{p}_k = [p_k, H]$$

and show that for any function $f(q_i, p_i, t)$

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + [f, H]$$

where H is a Hamiltonian.