EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

SECOND EXAMINATION IN SCIENCE 2008/2009

SECOND SEMESTER (Sept/Oct., 2010)

MT 217 - MATHEMATICAL MODELING

Answer all questions **Time: Two hours**

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1. Describe the steps involved in a mathematical model building process.

   A man eats a diet of 2500 calorie/day, 1200 of them go to basal metabolism (that is, get used up automatically). He spends approximately 16 calorie/day times his body weight (in Kg), in wight proportional exercise. Assume that the storage of calories as fat, 100% efficient and that 1 Kg fat contains 10000 calorie. Find how his weight varies with time.

2. Distinguish between exponential and logistic population growth. Give the equations for each.

   Consider the fish population, $P(t)$, in a fresh-water man-made pond . Assume that the initial population is set at 100 fish by stocking the pond with a combination of mature male and female fresh-water trout. Based on various environmental factors (food supply, water conditions, etc.), a reasonable approximation to the population dynamics for low population sizes suggests that the normalized fish birth and death rates are both inversely proportional to the square root of the population present at time t.

(a) Based on the above description, derive a mathematical model for the fish population and find a general solution for $P(t)$.

(b) Given that the initial population is 100 fish and that the measured population after 6 month is 169 fish, estimate how many fish will be in the pond after the first year.

(c) Using the same dynamics model, what will be the population size after five years? Based on your results, what can you say about the accuracy of the mathematical model for large population sizes?

3. Briefly describe the compartmental model with a single compartment, the lake, to model the changing concentration of pollution in a lake system.

A large tank has $0.8Kg$ of salt which is dissolved in $V_0 = 300$ liter water. Salt water flows in to the tank at a velocity 5 l/min. This water contains 0.006 Kg of salt per liter of water. The well-mixed salt water is drained from the tank with the same velocity. Give a mathematical formulation to calculate the amount of salt $M(t)$ (mass in Kg) in the tank at time t.

4. Suppose a G force and a A force are engaged in combat and G, is fighting guerilla style while A, is fighting in a conventional style. Instead of bing exposed, the guerilla fighters are hidden. Their opponents can not see them to fire directly at them. All that the conventional force knows is that the guerilla contingent is " over that field". So, the opposing force fires into that area and hopes for hit. The differential equation that represents this situation is given as follows:

$$\frac{dA}{dt} = -gG$$

$$\frac{dG}{dt} = -aAG.$$

(a) Explain the terms involved in these equations.

(b) For $a = 0.00002$, $g = 0.01$ and an initial value of A force $(A_0) = 100$, determine the smallest number of guerilla fighters that results in a victory for the guerilla force.

(c) Verify that if $a = 0.00002$ and $g = 0.01$, then an initial force of 40 guerilla fighters can fight an initial force of 200 conventional fighters to a draw.

(d) Find A as a function of G by solving the differential equation involving $\frac{dA}{dG}$. Show that

$$A = \sqrt{2\frac{g}{a}G + A_0^2 - 2\frac{g}{a}G_0} \text{ and } G = \frac{aA^2}{2g} + G_0 - \frac{aA_0^2}{2g}.$$