EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

FIRST EXAMINATION IN SCIENCE - 2009/2010

FIRST SEMESTER (June/July, 2011)

MT106 - TENSOR ANALYSIS

(REPEAT)

Answer all questions **Time : One hour**

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1. (a) Write the transformation equation for the following tensors:

   i. A^{ms}_{qr};

   ii. B^{pqr}_{mn};

   iii. C_{ijk}.

   (b) Define the terms symmetric and skew-symmetric tensors.

   i. If $ds^2 = g_{ij}\, dx^i\, dx^j$ is an invariant, then show that g_{ij} is a symmetric covariant tensor of rank two.

   ii. If A^{pq} and B_{rs} are skew-symmetric tensors, then show that $C^{pq}_{rs} = A^{pq}B_{rs}$ is a symmetric tensor.

   (c) The covariant components of a tensor in rectangular co-ordinate system are yz, 3, $2x+y$. Find its covariant components in cylindrical co-ordinates $(\rho,\ \phi,\ z)$.

2. (a) Define the Christoffel's symbols of the first and second kind.

(b) Determine the Christoffel's symbols of the second kind for the line element given by

$$ds^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2.$$

(c) With the usual notations, prove the following:

i. $\dfrac{\partial g_{pq}}{\partial x^m} = [pm, q] + [qm, p]$;

ii. $\dfrac{\partial g^{pq}}{\partial x^m} = -g^{pn}\Gamma^q_{mn} - g^{qn}\Gamma^p_{mn}$;

iii. $\dfrac{1}{2g}\dfrac{\partial g}{\partial x^m} = \Gamma^j_{jm}$.