EASTERN UNIVERSITY, SRI LANKA
THIRD EXAMINATION IN SCIENCE - 2004/2005
SECOND SEMESTER(Dec.,2008/Jan.,2009)
MT 310 - FLUID MECHANICS
(SPECIAL REPEAT)

Answer all Questions **Time: Two hours**

Q1. (a) If the velocity components of a fluid at (x, y, z) are

$$u = \frac{3x^2 - r^2}{r^5}, \quad v = \frac{3xy}{r^5}, \quad w = \frac{3xz}{r^5},$$

where $r^2 = x^2 + y^2 + z^2$, show that the motion described by $u, v,$ and w is possible. Furthermore, find the equation of stream lines. [60 marks]

(b) Show that

$$\left(\frac{x^2}{a^2}\right)\tan^2 t + \left(\frac{y^2}{b^2}\right)\cot^2 t = 1,$$

where a and b are constants, is a possible form for a boundary surface of a fluid. [40 marks]

Q2. Define the terms **Complex potential** and **Complex velocity** of an irrotational two dimensional motion of an inviscid fluid in z-plane. Find the complex potential of a uniform stream making an angle α with the real axis in z-plane.

Furthermore, if a circular cylinder of radius $\frac{1}{2}(a + b)$ is introduced into the above stream, find the complex potential in ζ-plane defined by the transformation

$$\zeta = z + \frac{c^2}{4z},$$

where c is a constant. [100 marks]

Q3. State and prove the theorem of Blasius and use it to find the force acting on cylinder due to a doublet.

[100 mark

Q4. (a) If a solid boundary of a large spherical surface contains fluid in motion an encloses closed surfaces $S_m, m = 1, \ldots, k$, write down the equation for th kinetic energy of the moving fluid when it is at rest at infinity. [15 mark

(b) If the velocity potential ϕ of a such fluid described in part (a) satisfies th Laplace's equation $\nabla^2\phi = 0$ and $\dfrac{\partial \phi}{\partial n}$ is a given function on $S_m, m = 1, \ldots,$ show that ϕ is determined uniquely throughout a finite region. [35 mark

(c) Suppose a puff of hot gas rises through still air and it takes a roughly spherica shape. The air is sucked into the gas near the rear of the sphere which ha fixed centre and radius a at time t. The mathematical model is given by

$$\nabla^2\phi = 0, \quad r > a,$$

$$\phi \simeq Ur\cos\theta \quad \text{as } r \longrightarrow \infty$$

with boundary condition

$$\left(\frac{\partial \phi}{\partial r}\right)_{r=a} = \frac{1}{5}\, v(1 - 3\cos\theta - 3\cos^2\theta),$$

where ϕ is the velocity potential, U is a constant velocity of the air and v is a constant. For the symmetry about the line $\theta = 0$, show that

$$\phi = Ur\cos\theta + \frac{Ua^3}{2r^2}\cos\theta.$$

What is the significance of the above result?

In spherical polar coordinates (r, θ, ψ),

$$\nabla^2 \equiv \frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\left(\frac{\partial^2}{\partial\psi^2}\right)$$

and you may use the solution of $\nabla^2\phi = 0$ without proof. [50 marks]