

EASTERN UNIVERSITY, SRI LANKA

THIRD EXAMINATION IN SCIENCE

(2002/03 & 2002/03(A))

SECOND SEMESTER (Apr./May.'2004)

Repeat

MT 310 - FLUID MECHANICS

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Answer all questions

Time: Two hours

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1. With the usual notation, derive the continuity equation for a fluid flow in the form

$$\frac{d\rho}{dt} + \rho \operatorname{div} \mathbf{q} = 0$$

where  $\frac{d}{dt}$  denotes the differentiation following a fluid particle.

Using the relations  $x = r \cos \theta$ ,  $y = r \sin \theta$ , express the vector

$$\mathbf{q} = \frac{c(-y\mathbf{i} + x\mathbf{j})}{x^2 + y^2},$$

in cylindrical polar coordinates  $(r, \theta, z)$ . Hence show that

- the motion of an incompressible fluid is possible, with velocity  $\mathbf{q}$ , and that the streamlines form a family of circles with centers on the  $oz$ - axis.
- this motion is irrotational with velocity potential  $\phi = -c\theta$
- streamlines intersect equipotential surfaces orthogonally
- the circulation of velocity around any curve in the  $oxy$ - plane is  $2\pi c$ .

2. With the usual notation, derive the equation of motion of an inviscid fluid in the form

$$\frac{dq}{dt} = \mathbf{F} - \frac{1}{\rho} \nabla p$$

A sphere of radius  $R(t)$  whose center is at rest, vibrates radially in an infinite incompressible liquid of constant density  $\rho$ .

The liquid, which is under no external body force, extends to infinity where it is at rest. Show that the motion of the liquid is irrotational with velocity potential

$$\phi = \frac{R^2 \dot{R}}{r} \quad \text{where} \quad \dot{R} = \frac{dR}{dt}$$

If the pressure at infinity is  $p_\infty$ , show that the pressure at the surface of the sphere ( $r = R$ ), at time  $t$ , is

$$p = p_\infty + \rho \left[ R \ddot{R} + \frac{3}{2} \dot{R}^2 \right]$$

If  $R = a + b \sin nt$  where  $a, b, n$  are constants such that  $a > b$ , show that in order that there is no cavitation  $p_\infty \geq \rho n^2 b(a + b)$ .

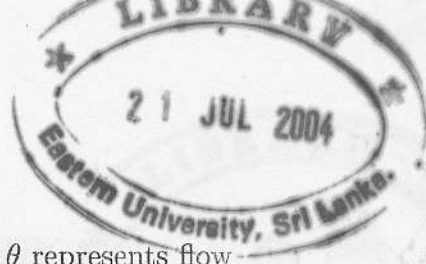
3. State and prove the Circle theorem for an irrotational two-dimensional flow of an incompressible inviscid fluid moving parallel to  $xy$ - plane

A two dimensional source of strength  $m$  is placed at a point

$C(z = c)$  outside a fixed circular boundary of centre  $O$  and radius  $a$

By finding the image system or otherwise, find the complex velocity at any point  $P(z)$  where  $|z| \geq a$ .

Show that the magnitude of the velocity is  $\frac{m \cdot AP \cdot BP}{OP \cdot CP \cdot DP}$ , where  $A, B$  are the points where  $OC$  cuts the circle and  $D$  is the inverse point of  $C$ .



4. Prove that the velocity potential  $\phi = U \left( r + \frac{a^2}{r} \right) \cos \theta$  represents flow past an infinite circular cylinder,  $r = a$ , fixed with its axis ( along  $Oz$  - axis) perpendicular to a uniform stream  $U$ , moving in the direction  $\theta = \pi$ , where  $(r, \theta, z)$  denotes cylindrical co-ordinates.

The pressure at infinity being given, calculate the resultant fluid thrust per unit length on half the cylinder lying on one side of a plane through the axis and parallel to the stream.