

EASTERN UNIVERSITY, SRI LANKA

THIRD EXAMINATION IN SCIENCE

(2002/03 & 2002/03(A))

SECOND SEMESTER (Apr./May '2004)

MT 310 - FLUID MECHANICS



Answer all questions

Time: Two hours

1. With the usual notation, derive the continuity equation for a fluid flow in the form

$$\frac{d\rho}{dt} + \rho \operatorname{div} \mathbf{q} = 0$$

where $\frac{d}{dt}$ denotes the differentiation following a fluid particle.

Using the relations $x = r \cos \theta$, $y = r \sin \theta$, express the vector

$$\mathbf{q} = \frac{c(-y\mathbf{i} + x\mathbf{j})}{x^2 + y^2},$$

in cylindrical polar coordinates (r, θ, z) . Hence show that

- (a) the motion of an incompressible fluid is possible, with velocity \mathbf{q} , and that the streamlines form a family of circles with centers on the oz - axis.
- (b) this motion is irrotational with velocity potential $\phi = -c\theta$
- (c) streamlines intersect equipotential surfaces orthogonally
- (d) the circulation of velocity around any curve in the oxy - plane is $2\pi c$.

2. With the usual notation, derive the equation of motion of an inviscid fluid in the form

$$\frac{d\mathbf{q}}{dt} = \mathbf{F} - \frac{1}{\rho} \nabla p$$

A sphere of radius $R(t)$ whose center is at rest, vibrates radially in an infinite incompressible liquid of constant density ρ .

The liquid, which is under no external body force, extends to infinity, where it is at rest. Show that the motion of the liquid is irrotational with velocity potential

$$\phi = \frac{R^2 \dot{R}}{r} \quad \text{where } \dot{R} = \frac{dR}{dt}.$$

If the pressure at infinity is p_∞ , show that the pressure at the surface of the sphere ($r = R$), at time t , is

$$p = p_\infty + \rho \left[R\ddot{R} + \frac{3}{2}\dot{R}^2 \right].$$

If $R = a + b \sin nt$ where a, b, n are constants such that $a > b$, show that in order that there is no cavitation $p_\infty \geq \rho n^2 b(a + b)$.

3. State and prove the Circle theorem for an irrotational two-dimensional flow of an incompressible inviscid fluid moving parallel to xy - plane.

A two dimensional source of strength m is placed at a point

$C(z = c)$ outside a fixed circular boundary of centre O and radius a .

By finding the image system or otherwise, find the complex velocity at any point $P(z)$ where $|z| \geq a$.

Show that the magnitude of the velocity is $\frac{m \cdot AP \cdot BP}{OP \cdot CP \cdot DP}$, where A, B are the points where OC cuts the circle and D is the inverse point of C .

4. For an incompressible fluid in irrotational motion under conservative forces obtain the pressure equation

$$\frac{p}{\rho} + \frac{1}{2} \mathbf{q}^2 + \Omega - \frac{\partial \phi}{\partial t} = f(t).$$

with the usual notation.



A solid sphere of radius a , uniform density σ moves in a straight line with velocity $U(t)$ through an infinite volume of liquid with uniform density ρ which is at rest at infinity. Show that

$$\frac{dU}{dt} = \frac{F}{\left(M + \frac{M'}{2}\right)},$$

where F is the external force acting on the sphere and $M = \frac{4}{3}\pi a^3 \sigma$ and $M' = \frac{4}{3}\pi a^3 \rho$.

Hence deduce that acceleration of a sphere falling vertically in an infinite fluid which is at rest at infinity is

$$\frac{g(\sigma - \rho)}{\left(\sigma + \frac{\rho}{2}\right)}.$$