02 JUN 2010
Eastern University, Sri Lanka.

EASTERN UNIVERSITY, SRI LANKA
DEPARTMENT OF MATHEMATICS
SECOND YEAR EXAMINATION IN SCIENCE -2008/2009
FIRST SEMESTER (Feb./Mar.,2010)
MT 203 - EIGEN SPACE AND QUADRATIC FORMS (PROPER/REPEAT)

Answer all Questions | Time: Two hours

1. Define the terms *eigenvalue* and *eigenvector* of a linear transformation. [10 marks]

   (a) (i) Prove that eigen vectors that corresponding to distinct eigen values of a linear transformation $T : V \to V$ are linearly independent, where V is a vector space. [30 marks]

   (ii) If A is an $n \times n$ real matrix and λ is an eigen value of the real symmetric matrix $(I_n + A^T A)$ then show that $\lambda \geq 1$, where I_n is the $n \times n$ identity matrix. [20 marks]

   (b) Let

$$A = \begin{pmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 3 \end{pmatrix}.$$

   Find a non-singular matrix P such that $P^{-1}AP$ is diagonal. [40 marks]

2. Define the term *minimum polynomial* of a square matrix. [10 marks] 4.

(a) Prove the followings:

(i) The characteristic polynomial of an $n-$square matrix A always divides the n^{th} power of its minimum polynomial. [30 marks]

(ii) The characteristic polynomial and the minimum polynomial of an $n-$square matrix A have the same irreducible factors. [20 marks]

(b) State the *Cayley Hamilton* theorem.

Find the minimum polynomial of the square matrix

$$\begin{pmatrix} 2 & 5 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 4 & 2 & 0 \\ 0 & 0 & 3 & 5 & 0 \\ 0 & 0 & 0 & 0 & 7 \end{pmatrix}.$$

[40 marks]

3. (a) Let λ_1 and λ_2 be two distinct roots of the equation $|A - \lambda B| = 0$, where A and B are real symmetric matrices and let u_1 and u_2 be two vectors satisfying

$$(A - \lambda_i B)u_i = 0 \quad \text{for } i = 1, 2.$$

Prove that $u_1^T B u_2 = 0$. [30 marks]

(b) Simultaneously diagonalize the following quadratic forms

$$\phi_1 = 3x_1^2 + 6x_2^2 + 6x_3^2 + 8x_1x_2 + 8x_1x_3 + 10x_2x_3,$$

$$\phi_2 = 2x_1^2 + 11x_2^2 + 3x_3^2 + 12x_1x_2 + 4x_1x_3 + 14x_2x_3.$$

[70 marks]

4. What is meant by an *inner product* on a vector space. [10 marks]

(a) Varify that the function $< .,. >$ defined by

$$< x, y >= \sum_{i=1}^{n} x_i \bar{y}_i, \quad x, y \in \mathbb{C}^n$$

is an inner product on $\mathbb{C}^n$. [25 marks]

(b) State and prove the *Cauchy Schwars* inequality. [25 marks]

(c) State the *Gram Schmidt process.*

Find an orthonomal basis of the subspace W of $\mathbb{C}^3$ spanned by $v_1 = (1, i, 0)$ and $v_2 = (1, 2, 1 - i)$. [40 marks]

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