

Answer all Questions

Time: Two hours

LIBRARD

0 2 JUN 2010

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- (a) For a five digit decimal floating point number, define the terms mantissa and characteristic. [20 marks]
 - (b) Explain what is meant by underflow and overflow in a calculation.

[15 marks]

(c) Describe the truncation error when $\sin x$ is approximated by x, where

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

[20 marks]

- (d) Suppose $a = 0.471 \times 10^{-2}$ and $b = -0.185 \times 10^{-4}$, use 3 digit floating point arithmetic to compute a + b and a - b and find the rounding error in each case. Hence find the rounding error in calculating $a^2 - b^2$. [25 marks]
- (e) For a binary computer working with 4 digit in the mantissa, floating point arithmetic, find the bound of the relative round-off error. [20 marks]

- 2. (a) Using bisection method find a root of the equation $t^3 + t 9 = 0$ correct to or decimal place. [25 mark
 - (b) Define fixed points of the iteration.

$$s_{n+1} = s_n + \lambda (s_n^3 + s - 9).$$

Find an approximate range of values of the constant λ such that the iteratic converges to the fixed point near 2. [20 mark

(c) Find the order of a given method described by an equation

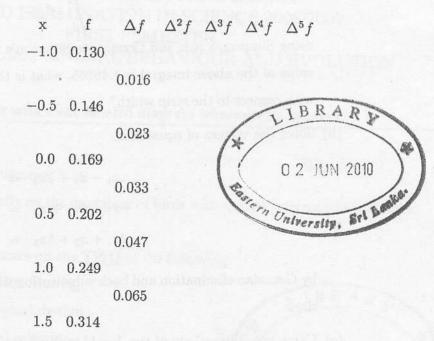
$$\epsilon_{r+1} - \epsilon_r = -\frac{2\epsilon_r f'(\alpha) + \epsilon_r^2 f''(\alpha)}{2[f'(\alpha) + \epsilon_r f''(\alpha)]},$$

[25 mark

where ϵ_r is the error in approximation after the r^{th} iteration.

(d) A calculator is defective: it can only add, subtract and multiply. Using the defective calculator and the Newton-Raphson method find an iterative formut to find $\frac{1}{a}$, where *a* is real number. Hence find a sequence of approximate value to $\frac{1}{1.37}$ correct to 4 decimal places. [30 mark]

3. (a) Complete the finite difference table for the function f(x) given by



Use a Newton's forward formula to find an approximation to f(-0.3) correct to 3 decimal places. [35 marks]

(b) Using the table given in (a) and a quadratic Lagrange's interpolation, find an approximation z to f'(-0.3). Further, using the error term in Lagrange interpolation find an expression for z - f'(x) by assuming that the third derivative of f(x) is constant in the interval you have chosen. [40 marks]

(c) The error in Simpson's rule for approximating the integral $I = \int_a^b f(x) dx$ is

$$-\frac{(b-a)^5}{2880}f^{(4)}(\xi), \quad a < \xi < b.$$

Show that the error in the Composite Simpson's rule for integrating f(x) from a to b is

$$-\frac{(b-a)}{180}h^4 f^{(4)}(\theta), \quad a < \theta < b.$$

(Assume that the fourth derivative of f(x) is continuous in [a, b])

[25 marks]

4. (a) Estimate

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 $\int_0^1 e^{x^2} dx$

using Simpson's rule and Composite Simpson's rule with 2 strips. If the exvalue of the above integral is 1.46265, what is the rate of decrease of the exwith respect to the strip width? [35 mail]

(b) Solve the system of equations:

$$x_1 - x_2 + 2x_3 = 0$$

-x_1 + 4x_2 + x_3 = 3
$$2x_1 + x_2 + 5x_3 = 1$$

by Gaussian elimination and back substitution without interchanging the eq tions. [25 mar

(c) Carry out 2-iterations of the Jacobi method for solving

$$10x - y - 3z = 1$$

$$x + 10y - 2z = 0$$

$$3x + 2y + 10z = -2$$

with initial estimation $(x, y, z)^T = (0, 0, 0)^T$.

Describe briefly how the Gauss-Seidel method differs from the Jacobi meth according to the above problem.

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