



EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

SECOND EXAMINATION IN SCIENCE - 2008/2009

FIRST SEMESTER (Feb./Mar., 2010)

ST 201 - STATISTICAL INFERENCE - I

(REPEAT)

Answer all questions

Time : Two hours

1. (a) Determine the maximum likelihood estimates of the parameters for the random sample of size  $n$  from each population given below:
- Poisson ( $\theta$ )
  - Normal ( $\mu, \sigma^2$ )
  - Bernoulli with  $p$

- (b) A random sample of  $X_1, X_2, \dots, X_n$  is obtained from the distribution with the probability density

$$f(x) = \frac{3\alpha^3}{(\alpha + x)^4}; \quad x > 0$$

where  $\alpha > 0$  is the unknown parameter. Find  $\hat{\alpha}$ , by the method of moments.

2. (a) A random variable  $X$  is distributed  $N(\mu, 16)$ . Suppose we know that  $\mu$  can take one of the two values 2 and 4. A random sample of  $X_1, X_2, \dots, X_{20}$  is drawn from the distribution to test the simple null hypothesis  $H_0 : \mu = 2$  against the simple alternative hypothesis  $H_1 : \mu = 4$ . Find the best critical region  $C$  of size  $\alpha = 0.025$ .
- (b) Derive the likelihood ratio test of hypothesis  $H_0 : \mu = \mu_0$  against  $H_1 : \mu \neq \mu_0$ , where  $\mu$  is the mean of a normal random variable with variance 1. Assume that a sample of size  $n$  is available.

3. A random sample  $X_1, X_2, \dots, X_n$  is taken from a Poisson distribution with mean  $\lambda$  and it is required to estimate  $\theta = \lambda^2$ .

(a) Show that the sample mean  $\bar{X}$ , is a sufficient statistic for  $\theta$ .

(b) Evaluate  $E(\bar{X})$  and  $E(\bar{X}^2)$  and hence find an unbiased estimator of  $\theta$  based on  $\bar{X}$ .

(c) Find the Gramer-Rao lower bound for the variance of unbiased estimators of  $\theta$ .

(d) Find the efficiency of your estimator in the case  $n = 1$ .

4. The distribution of  $X_{(n)}$ , the largest of the  $n$  observations in a random sample from a population that is uniform on  $[0, \theta]$ .

(a) Show that  $X_{(n)}$  is a consistent estimate of  $\theta$ .

(b) Determine a multiple of  $\bar{X}$  that is unbiased and obtain its mean squared error.

(c) Determine a multiple of  $X_{(n)}$  that is unbiased and compute its mean squared error.

(d) What is your conclusions about these estimators?