



EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

THIRD EXAMINATION IN SCIENCE-2008/2009

FIRST SEMESTER (FEB., 2010)

MT 304 - GENERAL TOPOLOGY

Answer all questions

Time: Two hours

1. Define the following on a non-empty set X :

- topology on X ;
- limit point of a subset of X ;
- dense subset of X .

(a) Let τ be the class of subsets of \mathbb{N} , set of all natural numbers, consisting of ϕ and all subsets of \mathbb{N} of the form $E_n = \{n, n+1, n+2, \dots\}$, where $n \in \mathbb{N}$. Show that τ is a topology on \mathbb{N} . Write down the open sets containing the element $5 \in \mathbb{N}$.

Determine the following:

- i. limit points of the set $A = \{4, 13, 28, 37\}$;
 - ii. subsets E of \mathbb{N} for which $E' = \mathbb{N}$, where E' is the set of all limit points of E ;
 - iii. closed subsets of (\mathbb{N}, τ) ;
 - iv. closure of the sets $\{7, 24, 47, 85\}$ and $\{3, 6, 9, 12, \dots\}$;
 - v. dense subsets of \mathbb{N} .
- (b) Prove that a subset A of a topological space X is closed if and only if A contains each of its limit points.
Hence show that, $S = \{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots\}$ is not closed under the usual topology on \mathbb{R} , set of real numbers.

2. Define the terms *base* and *subbase* for a topological space.

(a) Let \mathbb{B} be a class of subsets of a non-empty set X . Then prove that \mathbb{B} is a base for some topology on X if and only if it satisfies the following properties:

(i) $X = \bigcup_{B \in \mathbb{B}} B$,

(ii) for any $B, B' \in \mathbb{B}$, if $p \in B \cap B'$ then there exists $B_p \in \mathbb{B}$ such that $p \in B_p \subseteq B \cap B'$.

(b) i. Let D be the discrete topology on $Y = \{a, b, c, d, e\}$. Find a subbase \mathbb{S} for D , which does not contain any singleton sets.

ii. Let \mathbb{S} be a subbase for a topology τ on X and A be a subset of X . Show that the class $\mathbb{S}_A = \{A \cap S : S \in \mathbb{S}\}$ is a subbase for the relative topology τ_A on A .

3. (a) What is meant by a function f from a topological space X to a topological space Y is *continuous at a point* $x_0 \in X$?

Let f be a function from a topological space (X, τ_1) into a topological space (Y, τ_2) . Prove the following:

i. f is continuous if and only if $f^{-1}(G)$ is open in X for each open set G in Y .

ii. if S is a subbase for τ_2 , then f is continuous if and only if

$$f^{-1}(A) \in \tau_1, \forall A \in S.$$

(b) Define *Frechet space* (T_1) and *Hausdorff space* (T_2).

i. Prove that every T_2 space is T_1 . Is the converse true? Justify your answer.

ii. Prove that a topological space (X, τ) is a T_1 space if and only if every singleton subset of X is closed.

4. Prove or disprove the following statements:

(a) continuous image of a compact set in a topological space is compact.

(b) in the usual topology on \mathbb{R} , the subset $(0, 1)$ is compact.

(c) the class of open intervals $A_n = \{(0, \frac{1}{n}) : n \in \mathbb{N}\}$ satisfies the finite intersection property and $\bigcap_{n \in \mathbb{N}} A_n = \phi$.

- (d) (X, τ) is a compact topological space if and only if for every class $\{F_i\}$ of closed subset of X , $\bigcap_i F_i = \phi$ implies $\{F_i\}$ contains a finite subclass $\{F_{i_1}, F_{i_2}, \dots, F_{i_m}\}$ with $F_{i_1} \cap F_{i_2} \cap \dots \cap F_{i_m} = \phi$.

Eastern University, Sri Lanka

Third Year First Semester Examination in Science

2008/2009 (February 2010)

CH 303 Electrochemistry

(Open)

Name of candidate

Time : 01 hour

Useful constants: $R = 8.314 \text{ J mol}^{-1} \text{ K}^{-1}$, $F = 96485 \text{ C mol}^{-1}$, $1.602 \times 10^{-19} \text{ C} = 0.0001 \text{ V}$

1) Define the following terms which refer to the properties of acids and bases.

- (a) Acid strength
- (b) Acid conductivity
- (c) Acid strength

(11 marks)

2) Calculate the molar strength and the mass activity coefficient of 0.01 M aqueous of $\text{H}_2\text{C}_2\text{O}_4$.

(20 marks)

3) Write the oxidation, reduction and the Debye-Hückel-Onsager equations and identify the terms in it.

(10 marks)

4) The dissociation constant of COOHCOOH at 25°C is $1.35 \times 10^{-2} \text{ mol dm}^{-3}$. The molar conductivity at 25°C of a 0.01 M solution of this acid is $72.2 \text{ S cm}^2 \text{ mol}^{-1}$. Calculate the molar conductivity at infinite dilution of CH_2COOH at 25°C .

(20 marks)

Total (60)