EASTERN UNIVERSITY, SRI LANKA

SECOND EXAMINATION IN SCIENCE (2000/2001)

(MAY'2001)

FIRST SEMESTER

MT 203 - EIGENSPACE & QUADRATIC FORMS

Answer all questions **Time : Two hours**

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1. Define the terms "eigenvalue" and "eigenvector" of a square matrix.

   (a) Show that if $P^{-1}AP = D$ then for any positive integer k, $P^{-1}A^kP = D^k$, where A, P and D are square matrices with same order such that P is non singular and D is diagonal.

   (b) Let A be a matrix of order n such that $A^2 = I$. Show that every eigenvalue of A is 1 or -1. Deduce that if $\mathbf{tr(A) = n}$, then $A = I$.

   (c) Let $A = \begin{pmatrix} 4 & 4 & 4 \\ 6 & 6 & 5 \\ -6 & -6 & -5 \end{pmatrix}$.

   Find a non - singular matrix P such that $P^{-1}AP$ is diagonal. Hence find a matrix B such that $B^2 = A$.

2. (a) Define the terms "symmetric" and "positive definite" as applied to a real square matrix.

    i. Prove that a square matrix A is positive definite if and only if all the eigenvalues are positive.

    ii. Prove that if A is a symmetric matrix then eigenvectors corresponding to the distinct eigenvalues are orthogonal.

   (b) i. State the Gram - Schmidt process.

    ii. Find the orthonormal set for span of S in R^4, where $S = \{(1, 0, -1, 0)^T, (0, 1, 2, 1)^T, (2, 1, 1, 0)^T\}$.

3. Define the term "minimum polynomial" of a square matrix.

   (a) State and prove the Cayley-Hamilton theorem.

   (b) Prove that, for any square matrix A the minimum polynomial exists and is unique.

   (c) Find the minimum polynomial of the matrix A given by,

$$A = \begin{pmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -2 & 4 \end{pmatrix}.$$

4. (a) Find an orthogonal transformation which reduces the following quadratic form to a diagonal form

$$x_1^2 + 2x_2^2 + 3x_3^2 - 4x_1x_2 - 4x_2x_3.$$

(b) Simultaneously diadonalize the following pair of quadratic forms:

$$3x_1^2 + 6x_2^2 + 6x_3^2 + 8x_1x_2 + 8x_1x_3 + 10x_2x_3;$$

$$2x_1^2 + 11x_2^2 + 3x_3^2 + 12x_1x_2 + 4x_1x_3 + 14x_2x_3.$$