(PROPER \& REPEAT)

## COM 1114 - BUSINESS MATHEMATICS AND STATISTICS

1. A) Simplify the following expressions to the lowest terms.
i) $\frac{2(a+b)^{-1}-5(a-b)^{-1}}{4\left(a^{2}-b^{2}\right)^{-1}}$
ii) $\frac{3}{x+y}+\frac{2 x^{2}-2 x y+4 x-4 y}{4 x+8} \div \frac{y^{2}-x^{2}}{2 y}$
iii) $\frac{\left(2^{2 n}-3.2^{2 n-2}\right)\left(3^{n}-2.3^{n-2}\right)}{3^{n-4}\left(4^{n+3}-2^{2 n}\right)}$
iv) $\frac{\frac{5}{a+2}-\frac{1}{a-2}}{\frac{3}{2+a}+\frac{6}{2-a}}$
B) Factor the following expressions completely.
i) $\quad x^{4 n}-16$
ii) $\quad x^{2}-4 x+4-y^{2}$
iii) $\quad x^{6}+6 x^{3}-16$
iv) $\quad 36 x y^{2}-48 x y z^{2}+16 x z^{4}$
C) Solve the following equations.
i) $\frac{x}{2 x^{2}+3 x-2}-\frac{1}{2 x}=\frac{3}{x^{2}+2 x}$
ii) $\quad x^{2 / 3}-5 x^{1 / 3}+6=0$
iii) $\quad x-\sqrt{4-3 x}=-8$
iv) $\quad 2^{3 x-2}=16$
2. A) i)

$$
\text { If } \frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}}=\frac{1}{2} \text { find the value of } \frac{a^{2}+a b+b^{2}}{a^{2}-a b+b^{2}}
$$

ii)

$$
\text { If } a^{b}=b^{a} \text { show that }\left(\frac{a}{b}\right)^{a / b}=a^{\left(a / b^{-1}\right)}
$$

B) Of the equations $p+3 x=39$ and $p=9 x+9$, one is a supply function of a product and the other is a demand function of the product, where $p$ is the price of the product and $x$ is the quantity produced.
i) Sketch the two equations on the same axes.
ii) Label the demand and supply equations on the graph and give reasons for your choice.
iii) Find the equilibrium price and quantity.
C) The circulation of a newspaper is increasing at a constant rate. Three months ago circulation was 3200 . Today it is 4400 .
i) Express the circulation in terms of time.
ii) What will be the circulation in 2 months from today?
03. A) i) Explain what is meant by the statement "The simple events that constitute a sample space are mutually exclusive and exhaustive".
ii) An investor has asked his stock broker to rate three stocks A, B and C and list them in the order in which he recommend them. Consider the following events:
$L$ : Sock A doesn't receive the lowest rating
$M$ : Stock B doesn't receive the lowest rating
$N$ : Stock C receives the highest rating
a. Define the random experiment and list the simple events in the sample space
b. List the simple events in each of the events $L, M$, and $N$
c. List the simple belonging to each of the following events: ( $L$ or $M$ ), ( $L$ and $M$ ) and $\bar{N}$
d. Identify a pair of exhaustive events among $L, M$, and $N$.
iii) A store manager has a cross - classified sample of 250 customer purchases, as shown in the following table.

| Size of purchase | Method of payment |  |
| :--- | :---: | :---: |
|  | Cash | Credit card |
| Under Rs. 20 | 51 | 31 |
| Rs. 20 or More | 65 | 103 |

a) What is the probability that the customer selected paid by credit card?
b) What is the probability that the customer selected made a purchase of under Rs. 20 ?
c) Are the events "payment by cash" and "purchase of under Rs.20" mutually exclusive? Explain.
d) Are the events "payment by cash" and "purchase of under Rs.20" independent? Explain.
B) i) The probability that a contractor will get a plumbing contract is $2 / 3$ and probability that he will not get an electric contract is $5 / 9$. If the probability of getting at least one contract is $4 / 5$, what is the probability that he will get both?
ii) Suppose that a day's production schedule calls for 9000 items. Three machines A, B and C with a daily production capacity of 4000 have the probability that an item is defective on them as 1,2 and 4 percent respectively. On a given day 4000 items were produced on $A, 4000$ on B and 1000 on C. one item is selected and found defective. What is the probability that it was produced on C ?
(20 Marks)
04. A) i) If $A=\left(\begin{array}{ll}1 & -1 \\ 2 & -1\end{array}\right), B=\left(\begin{array}{rr}a & 1 \\ b & -1\end{array}\right)$ and $(A+B)^{2}=A^{2}+B^{2}$, find the value of $a$ and $b$.
ii) A manufacturer produces 2 types of products X and Y . Each product is first processed in a machine $M_{1}$ and then sent to another machine $M_{2}$ for finishing. Each unit of X requires 20 minutes time in $M_{1}$ and 10 minutes time on $M_{2}$ while the corresponding times for Y are 10 minutes on $M_{1}$ and 20 minutes on $M_{2}$. The total time available on each machine is 600 minutes. Calculate the number of units of X and Y by constructing a matrix equation of the form $\mathrm{A} X=\mathrm{B}$ and then solving by matrix inversion method.
B) The data given below show the daily output of a certain product for a month.

| Output <br> (Kg) | $1000-2000$ | $2000-3000$ | $3000-4000$ | $4000-5000$ | $5000-6000$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| No of Days | 20 | 15 | 10 | 25 | 10 |

i) Draw a histogram \& polygon for this distribution
ii) Calculate the following
a) Mean
b) Median
c) Mode
d) Standard deviation
e) Coefficient of variation
iii) Comment on the shape of the distribution.
05. A) Differentiate the following with respect to $x$.
i) $y=\frac{e^{\left(x^{2}+1\right)}}{\log x}$
ii) $\quad y=\sqrt{(x-1)^{5}(6 x-5)}$
B) If $y=10 \log \left(15-u^{2}\right)$ and $u=x^{2}-2 x+5$ find $\frac{d y}{d x}$.
C) A manufacturer has developed a new design for solar collection panels. The demand function for the panels has been estimated as,

$$
p=500-0.005 q
$$

Where $q$ equals the number of units demanded each year and $p$ equals the price in dollars. The total cost of producing $q$ panels is estimated as,

$$
c=150000+100 q+0.003 q^{2}
$$

i) Find revenue function in terms of $q$.
ii) Formulate the profit function.
iii) Determine the number of units $q$ that should be produced to maximize annual profit.
iv) Determine the price that should be charged for each panel to generate a demand equal to the answer in part (iii).
v) Determine the maximum annual profit.

