

## EASTERN UNIVERSITY, SRI LANKA

## **DEPARTMENT OF MATHEMATICS**

EXTERNAL DEGREE EXAMINATION IN SCIENCE - 2008/2009

FIRST YEAR SECOND SEMESTER (Jan./Mar., 2011)

EXTMT 104 - DIFFERENTIAL EQUATIONS AND FOURIER SERIES

Answer all questions

Time: Three hours

1, (a) State the necessary and sufficient condition for the differential equation

$$M(x,y)dx + N(x,y)dy = 0$$

to be exact.

Hence solve the following equation

$$\left(3x^4y^2 - \frac{1}{y}\right)\frac{dy}{dx} + 4x^3y^3 + \frac{1}{x} = 0.$$

(b) Find the general solution of the following ordinary differential equations:

i. 
$$(1 + e^{x/y}) dx + e^{x/y} \left(1 - \frac{x}{y}\right) dy = 0;$$

ii. 
$$x \frac{dy}{dx} + y = y^2 \log x$$
.

- 2. (a) If  $F(D) = \sum_{i=0}^{n} p_i D^i$ , where  $D = \frac{d}{dx}$  and  $p_i$ , i = 1, 2, ..., n, are constants with  $p_0 \neq 0$ . Prove the following formulas:
  - i.  $\frac{1}{F(D)}e^{\alpha x} = \frac{1}{F(\alpha)}e^{\alpha x}$ , where  $\alpha$  is a constant and  $F(\alpha) \neq 0$ ;
  - ii.  $\frac{1}{F(D)}e^{\alpha x}V = e^{\alpha x}\frac{1}{F(D+\alpha)}e^{\alpha x}V$ , where V is a function of x.
  - (b) Find the general solution of the following ordinary differential equations by using the results in (a).
    - i.  $(D^3 3D 2)y = 540x^3 e^{-x}$ ;
    - ii.  $(D^3 + 4D^2 + 4D)y = 8e^{-2x}$ .
- 3. (a) Let  $x = e^t$ . Show that

$$x D y \equiv \mathcal{D} y$$
,

and

$$x^2 D^2 y \equiv (\mathcal{D}^2 - \mathcal{D})y.$$

where  $\mathcal{D} \equiv \frac{d}{dt}$ .

Use the above results to find the general solution of the following ordinary differential equation

$$(x^2D^2 - 3xD + 4)y = x + x^2 \ln x.$$

(b) Solve the following simultaneous ordinary differential equations:

$$(5D+4)y - (2D+1)z = e^{-x}.$$
$$(D+8)y - 3z = 5e^{-x}.$$

4. Use the method of Frobenius to obtain two linearly independent solutions in series for the following ordinary differential equation

$$4x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 7y = 0.$$

5. (a) Write down the condition of integrability of the total differential equation

$$P(x, y, z)dx + Q(x, y, z)dy + R(x, y, z)dz = 0.$$

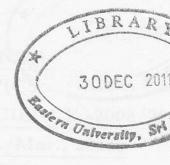
Solve the differential equation

$$y^2 dx - z dy + y dz = 0.$$

(b) Solve the following differential equations:

i. 
$$\frac{dx}{z(x+y)} = \frac{dy}{z(x-y)} = \frac{dz}{x^2 + y^2};$$

ii. 
$$\frac{dx}{xy} = \frac{dy}{y^2} = \frac{dz}{zxy - 2x^2}.$$



(c) Apply Charpit's method or otherwise to find the complete and the singular solution of the non-linear partial differential equation

$$2z + p^2 + qy + 2y^2 = 0.$$

(d) Solve the non-linear partial differential equation

$$pz - q^2 = 1.$$

You may assume that z = F(x + ay) = F(u), where u = x + ay and a is arbitrary constant.

6. (a) Obtain the Fourier series expansion of

$$f(x) = \begin{cases} 2x & \text{when} \quad 0 \leqslant x < 3, \\ 0 & \text{when} \quad -3 < x < 0. \end{cases}$$

- (b) Find the finite Fourier sine transform and the finite Fourier cosine transform of  $\frac{\partial u}{\partial x}$  and  $\frac{\partial^2 u}{\partial x^2}$ , where u is a function of x and t for  $0 < x < l, \quad t > 0$ .
- (c) Use part (b) to show that the solution of the partial differential equation

$$\frac{\partial V}{\partial t} = \frac{\partial^2 V}{\partial x^2},$$

subject to the boundary condition:

V(0,t) = 0, V(4,t) = 0, V(x,0) = 2x, where 0 < x < 4, t > 0, is

$$V(x,t) = \frac{-16}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} e^{\frac{-n^2 \pi^2 t}{16}} \cos n\pi \sin \frac{n\pi x}{4}.$$