

2. (a) Prove the followings:

i. the rate of change of total angular momentum \underline{H} about a point O is equal to the total moment of the external forces about O ,

ii. $\underline{H} = \underline{r}_G \wedge M \underline{v}_G + \underline{H}_G$,

where \underline{H}_G is the angular momentum about the centre of mass and \underline{v}_G is the velocity of the centre of mass.

(b) A uniform sphere of mass M and radius a is released from rest on a plane inclined at an angle α to the horizontal. If the sphere rolls down without slipping, show that the acceleration of the center of the sphere is constant and is equal to $\frac{5}{7} g \sin \alpha$.

3. (a) With the usual notations, state the Euler's equation for the motion of a rigid body with one point fixed.

A body moves about a point O under no forces. The principal moments of inertia at O being $3A$, $5A$ and $6A$. Initially the angular velocity has components $\omega_1 = n$, $\omega_2 = 0$, $\omega_3 = n$ about the corresponding principal axes. Show that at any time t ,

$$\omega_2 = \frac{3n}{\sqrt{5}} \tan\left(\frac{nt}{\sqrt{5}}\right)$$

and that the body ultimately rotates about the mean axis.

4. (a) Define the Hamiltonian in terms of the Lagrangian.

Hence show that the Hamiltonian's equations are given by

$$\dot{q}_j = \frac{\partial H}{\partial P_j}, \quad \dot{P}_j = -\frac{\partial H}{\partial q_j},$$

when H does or does not contain the variable time t explicitly.

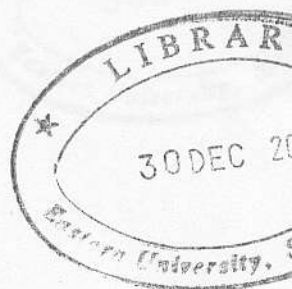
(b) If the Hamiltonian H is independent of time t explicitly, then prove that it is

i. a constant,

ii. equal to the total energy of the system.

(c) A particle moves in the xy plane under the influence of a central force depending only on its distance from the origin.

- i. Set up the Hamiltonian for the system.
- ii. Write the Hamiltonian's equations of motion.



5. (a) With the usual notations, derive Lagrange's equation for the impulsive motion from Lagrange's equation for a holonomic system in the following form

$$\Delta \left(\frac{\partial T}{\partial \dot{q}_j} \right) = S_j, \quad j = 1, 2, \dots, n.$$

(b) A square $ABCD$ formed by four equal rods, each of length $2l$ and mass m joined smoothly at their ends, rests on a smooth horizontal table. An impulse of magnitude I is applied to the vertex A in the direction of AD .

- i. Find the equation of motion of the frame.
- ii. Show that the kinetic energy of the square immediately after the application of impulse is $\frac{5I^2}{16m}$.

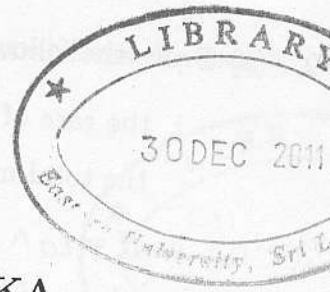
6. (a) Define the Poisson Bracket.

With the usual notations show that

$$\frac{dF}{dt} = [F, H] + \frac{\partial F}{\partial t}$$

for a function $F = F(p_j, q_j, t)$, $j = 1, 2, \dots, n$. Prove the Poisson's theorem that $[F, G]$ is a constant of motion when $F = F(p_j, q_j, t)$ and $G = G(p_j, q_j, t)$, $j = 1, 2, \dots, n$ are constant of motion.

(b) Find the frequency of oscillation of a particle of mass m which is moving along a line and is attached to spring whose other end is fixed at a point A at a distance l from the line.



EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

EXTERNAL DEGREE EXAMINATION IN SCIENCE - 2008/2009

THIRD YEAR, SECOND SEMESTER (Jan./Apr., 2010)

EXTMT 307 - CLASSICAL MECHANICS III

Answer all Questions

Time: Three hours

1. A projectile located at colatitude λ is fired with velocity v_0 in a southward direction at an angle α with the horizontal.

- (a) Find the position of the projectile after a time t .
- (b) Prove that after time t the projectile is deflected toward the east of the original vertical plane of motion by the amount

$$\frac{1}{3}\omega g \sin \lambda t^3 - \omega v_0 \cos(\alpha - \lambda)t^2$$

where ω is the rotation speed of the earth.

- (c) Prove that when the projectile returns to the horizontal, it will be at the distance

$$\frac{4\omega v_0^3 \sin^2 \alpha}{3g^2} \{3 \cos \alpha \cos \lambda + \sin \alpha \sin \lambda\}.$$