EASTERN UNIVERSITY, SRI LANKA

EXTERNAL DEGREE EXAMINATION IN SCIENCE

FIRST YEAR FIRST SEMESTER -2004/2005

(May/ Jun., 2008)

MT 106 - TENSOR CALCULUS

(Proper and Repeat)

Answer all questions **Time : One hour**

1. (a) Write the transformation equation for the following tensors:

   i. A^{ms}_{qr},

   ii. B^{pqr}_{mn},

   iii. C_{ijk}.

   (b) Let A^{pq}_{rst} be a tensor.

   i. Choose $p = t$ and show that A^{pq}_{rsp}, where the summation convention is employed, is a tensor. What is it rank?

   ii. Choose $p = t$ and $q = s$ and show similarly that A^{pq}_{rqp} is a tensor. What is its rank?

   (c) Find g and g^{jk} corresponding to the line element
   $ds^2 = 5(dx^1)^2 + 3(dx^2)^2 + 4(dx^3)^2 - 6dx^1dx^2 + 4dx^2dx^3$.

2. (a) Define the following:

i. Christoffel symbols of the first and second kind;

ii. Geodesic.

(b) Determine the Christoffel's symbol of second kind for the metric

$$ds^2 = d\rho^2 + \rho^2\, d\phi^2 + dz^2$$

and the corresponding differential equations for geodesic.

(c) With usual notations, prove the following:

i. $[p\,q,\, r] = g_{r\,s}\Gamma^{s}_{p\,q}$;

ii. $[p\,r,\, q] + [q\,r,\, p] = \dfrac{\partial g_{p\,q}}{\partial x^r}$;

iii. $\dfrac{\partial g^{p\,q}}{\partial x^r} + g^{p\,s}\Gamma^{q}_{r\,s} + g^{q\,s}\Gamma^{p}_{r\,s} = 0.$

Hence show that $g_{p\,q;\,r} = 0$.