EASTERN UNIVERSITY, SRI LANKA

SECOND EXAMINATION IN SCIENCE (1996/97)

(June/August' 2004)

EXTERNAL DEGREE

EXMT 207 & 209 - CLASSICAL MECHANICS II AND DIFFERENTIAL EQUATIONS & FOURIER SERIES

Answer four questions only selecting two questions from each section

Time : Two hours

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1. With the usual notations, obtain the following equations for a common catenary.

   (a) $s = C\tan\psi$,

   (b) $y = C\sec\psi$,

   (c) $T = \omega y$,

   (d) $y^2 = s^2 + c^2$.

   A uniform flexible chain of length l and weight per unit length w, rests in a vertical plane with length kl $(0 < k < 1)$ in contact with a smooth plane inclined at an angle α to the horizontal. Upper end of the chain is attached to a point P. Show that the tension at P is

$$wl\sqrt{1-k(2-k)\cos^2\alpha}$$

Find the horizontal and vertical distance of P from the lower end.

2. If S and M are shearing force and bending moment respectively at a point of uniformly loaded beam, then prove that

$$\frac{dS}{dx} = \omega, \quad \text{and} \quad \frac{dM}{dx} = -S\,,$$

where ω is the weight per unit length of the beam.

State the Bernoulli-Euler law of flexure.

A uniform elastic beam AB of length $3a$ and weight W is clamped horizontally at its ends, which are at the same horizontal level. Two concentrated loads W and $2W$ are placed at the points of trisection of the beam with smaller load near to A. Show that the reaction at A and B are $\frac{95W}{54}$ and $\frac{121W}{54}$ respectively. Find also the bending moment at each points.

3. With the usual notations, prove the Claypeyron's equation

$$M_1 a + 2M_2(a+b) + M_3 b = -\frac{W}{4}(a^3+b^3) + 6EI\left(\frac{y_a}{a} + \frac{y_b}{b}\right)$$

for the moment of a slightly elastic beam.

A uniform slightly elastic beam AD of length $4a$ and weight W rests on four supports which are in the same horizontal level. The supports are placed at the end points of the beam and at points B and C such that $AB = 2a, BC = a$ and $CD = a$. Show that the magnitude of the bending moments at B and C are $\frac{17Wa}{184}$ and $\frac{3Wa}{368}$ respectively. Find the ratio of the reactions at the four supports.

Section B

4. Obtain the solution of the differential equation

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - 1)y = 0$$ ← Centre

in series.

5. (a) State the necessary and sufficient condition for the equation $Pdx + Qdy + Rdz = 0$ (to be integrable), ← Centre

Where P, Q and R are functions of x, y, z.

Test the integrability of the differential equation

$$(2x^3y + 1)dx + x^4dy + x^2\tan z\, dz = 0$$ ← Centre

and solve this when it is integrable.

(b) Find the general solution of the following equations:

i. $\frac{dx}{y - z} = \frac{dy}{z - x} = \frac{dz}{y - x}$;

ii. $\frac{dx}{x(z^2 - y^2)} = \frac{dy}{y(x^2 - z^2)} = \frac{dz}{z(y^2 - x^2)}$.

(c) Find the complete solution and the singular solution of the following equations, if $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$.

i. $z = px + qy + 3p^{\frac{1}{3}}q^{\frac{1}{3}}$;

ii. $x^2p^2 + y^2q^2 = z$. (Hint : Use $X = \log x,\ Y = \log y$)

6. (a) Let $V = V(x,t)$ be a function such that V and $\frac{\partial V}{\partial x}$ each approaches to zero as $x \to \infty$.

Show that

$$\int_0^\infty \frac{\partial^2 V(x,t)}{\partial x^2} \sin \lambda x \, dx = \lambda V(0,t) - \lambda^2 U,$$

where $U = \int_0^\infty V(x,t) \sin \lambda x \, dx$.

(b) Use the sine transformations to show the solution of the partial differential equation $\frac{\partial^2 V}{\partial x^2} = \frac{\partial V}{\partial t}$ for $x > 0,\ t > 0$ satisfying the conditions,

i. $V(x,t) = \cos t$ when $x = 0$,

ii. $V(x,0) = 0$,

is given by

$$V(x,t) = \frac{2}{\pi} \int_0^\infty \left(\frac{\sin t + \lambda^2 \cos t - \lambda^2 e^{-\lambda^2 t}}{\lambda^4 + 1} \right) \lambda \sin \lambda x \, d\lambda.$$